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Probability

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Abstract:. *Probability are introduced as semiring algebraic structures that retain verious parcels of classical chances taking values in the real number interval [0, 1]. Compact chances and arbitrary variables with similar chances are substantially studied. Analogs of the Borel- Cantelli lemma and of the law of large figures are considered. New sundries of superposition of probability spaces and superposition of arbitrary variables arise on the base of the Cartesian product of abstract chances*

Keywords: probability

I. INTRODUCTION

In life, an action performed by us may yield to different results. Consider the simple example of throwing a six-faced cubic die. The uppermost face can show any one of the six numbers 1,2,3, 4, 5, 6. No one can say with certainty as to which number will show up at a particular time. Hence we have to estimate the chance that a particularnumber will come up. If the die is not a crocke dir, then obviously cach number has an equal chance of turning up. Since there are six poble results, we can say that eachnumber has '1 chance in 6 'of turning up. Such a chance is called "probability". In thischapter, we will learn various methods to calculate the probability for a given experiment. We will also make extensive use of Permutations and Combinations, sostudents a requested to revise those concepts before starting with this chapter.

Probability

The word probability is deduced from the word "probable.

When the outcome of any event is not certain, the theory of probability enters thescene .

For example, when a coin is tossed, whether it will show Head or Tail, is not certain, and we say that "probably it may show a Head or maybe a Tail"!

The theory of probability calculates the chances of occurrence of an outcome.

Consider the example of a coin, for a fair toss, it is anybody's guess that the chances of occurrence of a Head or a Tail are equal, ie, 50:50. However, this is just a common sense. The same common sense supported by theoretic principles, enfolds the fascinating world of probability theory in front of us. So let us learn and enjoy the concept called Probability!

We begin with Basic Terminology

Basic Terminology 1.Random Experiment:

Any action, whose result is uncertain, not pre-decided is called a random experimentor trial. Each result of the trial is called an outgrowth of the trial.

Sample Space:

The set of all possible outcomes of an experiment or trial is called the sample spaceof that experiment and is denoted by S. This will be Universal Set for the said experiment

we shall be considering only those experiments whose sample spaces are finite.Hence n(S), the number of elements of S is a natural number.

Let us study S for some experiments and find (S).

Consider the simple experiment of tossing a coin. Since the coin can be headtor tail at a random sample space

S = (H,T).

(S) = 2.

On the same lines, the tossing of a cubic die gives the sample spaceS=(1,2,3,4,5,6)

n(S)=6

If a coin is tossed followed by the throwing of a cubic die, the sample spacecontain 2x 6=12 outcomes given by s={(H,1),(H,2),(H,3),(H,4),(H,5),(H,6), (T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}n(S)=12

Note: We shall consider only those sample spaces in which each single outcome is equally likely therefore there is no reason to expect the occurrence of any one of the outcomes over any other. Such a sample space is the result of an unbiased experiment.

Events:

Consider the experiment of throwing a cubic die. Then s={1,2,3,4,5,6} is the sample space. This contains 6 outcomes. Now we may be interested in some " For example, we wish the number 6 to come up. This happening is the subset (6) of S,But if we want an even number to come up, this happening is the subset {2,4,6} of S. In general, any happening is a subset of S and is called an event.

Hence, an event E is any subset of the sample space S. E happens when one of the outcomes in E takes place, E does not happen if the outcome that actually takes place, does not belong to E As explained earlier, every sample point is equally likely to happen .If n(E)=1, i.e.if contains exactly one sample point then E is called an elementary event. Any event A, which contains more than one sample point, can be considered as the union of a number of elementary events. For example, the event E explained above, that of getting an even number from a simple throw of a die, is given by E={2,4,6} , which E={2} U (4) U (6). is

Impossible Event and Certain Events:

It is possible that event E={}, i.e. a null set Such an event is an impossible event and n (E)=0 in this case. The other extreme case is E S .Such an event is sure to occur no matter what is the outcome. In such a case, E is called a certain event Obviously n(E)=n(S) , in this case.

Equally Likely Events:

If each event has equal chance of appearance they are called Equally Likely events The theory of probability is based on the fact that, the events are equally likely. This is indicated by the terms, 'unbiased', fair', etc.

Some Standard Examples

1. Tossing of a coin/coins:

This is already discussed while describing a sample space.

2. Throwing of a die/dice:

When a single die is thrown, the outcomes are: S={1,2,3,4,5,6\} . n(S)=6.

i) If A is an event that an odd number comes up, then A={1,3,5}

n(A)=3.

ii) If B is the event that the number is divisible by 3, then B={3,6}

n(B)=2

iii) If C is an event that the number 7 comes up, then C is an impossible event and c={ }

n(C)=0

iv) If D is the event that a number less than 7 comes up, then D is a certain event and n(D)=n(S)=6.

The other standard example is of two dice, It is discussed in detail in the Illustrative Examples.

3. A Pack of Cards:

A pack consists of 52 cards, which are as follows: Red(26)

Face Cards : Jack, Queen and King are called face cardsThere are 12 face cards in a pack of 52 cards.

An Ace: A card bearing the number I is called an Ace. Thus, a pack contains 4 ace cards.

Consider an event of drawing a card from a pack Since each outcome has to be equally likely, we will always consider a’ well-shuffled' pack of cards.

Thus, if a card is drawn from a well-shuffled pack of cardswhich contains 52 cards, then S will contain each card, i.e. S will contain 52 sample points. n(S)=52

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i) If A is an event, that a red card is drawn Then, A will contain 26 red cards. n(A)=26.
ii) If B is the event that a club is drawn, then n(B)=13 since there are 13 clubs in a pack.
iii) Similarly, if C is an event that a king is drawn, then n(C)=4
However, if D is the event that a black king is drawn, then n(D)=2 , since there are two black kings, one of club and the other of spade
iv) The event that a rabbit appeared when a card is drawn from a pack of cards, is an impossible event (even though many magicians are able to do it!). Similarly, a card drawn is not blue' in colour, is a certain event
We have discussed some standard examples, their outcomes and events. Let us now combine and classify the events in the following:

4. Algebra of Events:

1. Union of Events: If A and B are two events of the sample space S then AUB (or A+B) is the event that 'either A or B (or both)' take place.
2. Intersection of Event: If A and B are two events of the sample space S then NB (or AB) is the event that both A and B take place.
3. Mutually Exclusive Events: Two events A and B of the sample space S are said to be utually exclusive if they cannot occur simultaneously. In such a case AB is a null set.
A n B is an impossible event.
4. Exhaustive Events: Two events A and B of the sample space S are said to be exhaustive if A\cup B=S, i.e. A UB contains all sample points. A\cup B is a certain event.
5. Complementary Events: Two events A and B are said to be complementary if:
i) A and B are mutually exclusive and
ii) A and B are exhaustive.
Note: It is clear that B is the event 'Not A'. B is also denoted by A.
We have, i) not (not A)=A . (A')'=A.
ii)A∩A'={} and AUA'=S
Independent Events: Two events A and B of the sample space S are said to be independe if the occurrence or non-occurrence of any one of the events has nothing to do with the occurrence non-occurrence of the other
eg :-Examination and Public Holidays are two dependent events whereas examinations rainfall are two independent events
Definition of Probability
Definition: If S is the finite sample space of an experiment and every outcome of equally likely and if E is an event (.. E C S)then the probability that E takes place is defined as :
P(E)=n(E)/n(s)
Note: Since S contains all possible outcomes of the experiment and E contains only the outcomes in which E happens, (outcomes favourable to E)_{i} we get,
P(E) =No of outcomes favourable to E/ No of all possible outcomes Results: The following results are direct consequences of the definition.
1. If E is an event of sample space S then 0≤ P(E)≤ 1.
Recall that the extreme cases ,P(E)=0 and P(E)=1 occur under the following Circumstances:
i) P(E)=0 ,if and only if E is an impossible event
ii) P(E)=1, if and only if E is a certain event.
2. If E is an event of sample space S and E' (or E) is the event that E does not happen, then P (E')=1-P(E).
EXAMPLE:-
Ex. 1. Three coins are tossed. Find the probability of getting:
i) 0 No Head   ii) Two Heads.
iii) At least Two Heads.   iv) Atmost Two Heads. Ans. Three coins are tossed. $n(S)=n=2^3=8$.
i) Let A be the event that no head shows up ,which means that, all tails show up .This is a single case (T,T,T)
n(A)=1 P(A)=n(A)\n(S) =1\8

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ii) Let B be the event that two heads show up.
This is when the situation is (H,H,T) or (H, T,H) or (T,H,H). n(B)=3
P(B)=n(B)\n(S) =3\8
iii) Let C be the event that at least two heads show upwhich means two heads of 3 heads show up.
The various cases are:
2 Heads = (H,H,T) , (H,T,H) , (T,H,H)
3 Heads (H,H,H) n(C)=3+1=4
Also, n(S) =8
P(C) =n(C)\n(S)=4\8 =1\2
iv) Let D be the event that atmost two heads show up .It means that, no head or 1 head or 2 heads show up.
The various cases are:
no Head= (T,T,T)
1 Head =(H,T,T)(T,H,T)(T,T,H)
2 Heads = (H,H,T) , (H,T,H) , (T,H,H) n(D)=1+3+3= 7
Also ,n(S)=8 P(D)=n(D)\n(S)=7\8
Ex. 2. A perfect cubic die is thrownFind the probability that: i)an even number comes up.
ii) a perfect square comes up.
Ans. Since a die can result in six outcomes,n (S)=6.
Observe that all
these outcomes are equally likely.

i) Let A be the event that an even number comes up. Then A ={2,4,6}.
n(A)=3.
P(A) =n(A)\n(S) =3\6 =1\2
ii) Let B be the event that a perfect square comes up. Since the only perfect squares in S are I and 4,
B = {1,4} . n(B)=2
P(A) =n(B)\n(S) =2\6 =1\3.
Ex. 3. Two perfect cubic dice are thrown. Find the probability that the sum of the numbers on their upper faces is at least 9.
Ans. When two perfect dice are thrown, the sample space S will consist of the following 36 sample points.
S={(1, 1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2, 1),(2, 2),(2,3),(2,4),(2,5),(2,6)
(3, 1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4, 1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5, 1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6, 1),(6, 2),(6,3),(6,4),(6,5),(6,6)}
where the first number in each bracket is on the uppermost face of the first die and the second is on the uppermost face of the second die .
n(S)=36
Let A be the event that the sum of these two numbers is at least 9. Since the sums of the two numbers can be any number between 2 and 12 (both inclusive), the required sums will be:
9, 10, 11 or 12.
A is the event that the sum of the numbers is 9, 10, 11 or 12. A contains the pairs:
Sum 9→ (3,6),(4,5),(5,4),(6,3)
Sum 10 → (4,6),(5,5),(6,4)
Sum 11 → (5,6),(6,5)
Sum 12→ (6,6)
n(A) 10.
P(A)=n(A)\n(s)=10\36 =5\18

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is the probability that the sum is at least 9.
Ex4)One card is drawn from a pack of well shuffled cards. Find the probability that, it is
i) a king,
ii) a face card,
iii) a spade,
iv) a spade or a king.
Ans. A pack of card has 52 cards in it. n(S)=52
1) Let A be the event that a king is drawnSince there are 4 kings in a pack, n(A)=4
P(A)= n(A)\n(s)=4\52 =1\13
ii) Let B be the event that a face card is drawn. Since there are 12 face cards in pack, n(B)=12
P(B)= n(B)\n(S)=12\52 =3\13
Therefore, the probability of drawing a face card is 3\13
iii) Let C be the event of drawing a spade. Since there are 13 spades in a pack,
n(C)=13
P(C))= n(C)\n(S)=13\52 =1\4
Therefore, the probability of drawing a spade card is 1\4
iv) Finally, let D be the event of drawing a spade or a king from the packObserve that, there are 13 spades and 4 kings in a pack.
Note the next crucial argument.
Of these 13 spades and 4 kings, one card, that of a spade-king is counted twice. It is accounted for, in spades, as well as in kings. Thus, the number of spades or kings in a pack is actually
13 + 4 - 1 = 16
Spades Kings King of Spade Hence n (D) = 16
kings king of spade
P(D))= n(D)\n(S)=16\52 =4\13
Therefore, the probability of a card, which is a spade or a king, is 4\13. Note: You may try finding the probability of a spade or a face card
The number of such cards is:
13 + 12 - 3 =22
Spades Face cards face cards of spadE
In the next example, we draw 2 cards instead of 1from a pack of well-shuffled cards. Note that, this is the case of choosing 2 cards out of 52 cards.
n(S)= 52^C2 =52.51\1.21
On the same lines, when 3 cards are drawn, n(S)=52^C 2and so on.

II. CONCLUSION

In conclusion, the probability is a tool that make easy to understand the world around us. By studying probability and statistics, you can better understand the chances of certain events occurring and make sounder decisions.The second section introduces the concept of complementary events--that is, events whose probabilities add up to 1People use probability and odds in many situations -- to decide what kind of weather to prepare for, whether or not to buy a stock, and how much to bet when gambling.