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Integration

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Abstract*: In mathematics, an integral is the continuous analog of a sum, which is used to calculate areas, volumes, and their generalizations. Integration, the process of computing an integral, is one of the two fundamental operations of calculus, the other being differentiation. Integration started as a method to solve problems in mathematics and physics, such as finding the area under a curve, or determining displacement from velocity. Today integration is used in a wide variety of scientific fields.The integrals enumerated here are called definite integrals, which can be interpreted as the signed area of the region in the plane that is bounded by the graph of a given function between two points in the real line. Conventionally, areas above the horizontal axis of the plane are positive while areas below are negative. Integrals also refer to the concept of an anti-derivative, a function whose derivative is the given function; in this case, they are also called indefinite integrals. The fundamental theorem of calculus relates definite integrals with differentiation and provides a method to compute the definite integral of a function when its anti-derivative is known; differentiation and integration are inverse operations..*

Keywords: integration

I. INTRODUCTION

Calculus is the gateway to advanced mathematical thinking. One of the reasons that calculus plays this role is that, to understand its content and possibilities, a shift in the way of thinking and doing mathematics is required. Although Newton and Leibniz are attributed with the discovery of calculus in the seventeenth century, the development of calculus started with the Greeks, and its content was not really formalized until the 1800's, due to what that shift in thinking and performing implies. Modern day calculus students must possess local fluency, as will be defined in the present study, in areas such as algebra, trigonometry and geometry, to be able to make the necessary connections that will permit success in applications, and understanding of underlying concepts; without these connections, the possibilities of using the powerful tools of calculus in a creative manner are reduced. The definition of mathematical fluency and its classification, as well as the method for detecting mathematical fluency using parameters employed in foreign language learning, is an important part of the - 1 - Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Present study. Without a method to detect where, in the process of learning, the problems lie, it will not be possible to correct them, or to design effective ways of transmitting knowledge. This study intends to shed light on aspects of the learning process related to applications of the integral, through the development of techniques for detecting mathematical fluency, based on the parameters of foreign language learning. The method itself transcends the particular case of integral calculus, and its validity for research in mathematics education in general is suggested. The motivation for conducting research on the process and concept of integration, and relating it to mathematical fluency, is multifold. The concept of mathematical fluency, as developed in this study, is the backbone of the theoretical and methodological aspects of this work. The concept of mathematical fluency is closely related to fluency in natural language, and language learning. On the other hand, the subject of calculus as a whole, and integration in particular, is one of those places in the mathematics curriculum where many students, previously successful, begin to find insurmountable difficulties. The questions that will be raised in the present study are related to the learning process, and how this is affected by what students face in the standard second calculus program, as well as their personal background. Another factor which will be examined, in terms of learning, is how the initial presentation of the definite integral as a means for calculating the "area under the curve" could, in itself, be the cause of a "cognitive obstacle" (Brousseau, 1997), when the student is required to broaden that particular interpretation to the - 2 - Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Calculation of volumes, arc lengths and applications to physical concepts. The second calculus course should be a

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natural and coherent continuation of the first in which, according to the standard curriculum content, the Fundamental Theorem of Calculus and the Riemann sum approach to integration are learned. There are studies (Thompson, 1994; Czarnoch, Loch, Prabhu, Vidakovic, 2001) which show that this objective has not been achieved, and the present study intends to further explore the question of continuity and coherence in a particular phase of the calculus sequence. The present study will refer to the part of the second calculus course that deals with applications of the integral in terms of calculating volumes, arc lengths, surface areas, work and moments. The study is strongly oriented towards the understanding of how learning, when applied to integral calculus, is affected by the dual nature of the integral symbol itself, which can be seen as an instruction to carry out an operational process, as well as the embodiment of a specific object which is produced by that process, representing the mathematical concept of accumulation. In addition, this study examines how feasible it is to use the powerful techniques of integral calculusin a creative way (to do modeling), as a scientist, economist, statistician, or engineer, if conceptual understanding of the integral, as representing accumulation, is not achieved.Instructional methods and content, while not themselves the subject of the present work, will be analyzed and reflected upon.

INTEGRAL

In general, the integral of a real-valued function *f*(*x*) with respect to a real variable *x* onan interval [*a*, *b*] is written as

$$\int_{b}^{a} f(x)dx$$

The integral sign ∫ represents integration. The symbol *dx*, called the differential of the variable *x*, indicates that the variable of integration is *x*. The function *f*(*x*) is called theintegrand, the points *a* and *b* are called the limits (or bounds) of integration, and the integral is said to be over the interval [*a*, *b*], called the interval of integration. A function is said to be *integrable* if its integral over its domain is finite. If limits are specified, the integral is called a definite integral.

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When the limits are omitted, as in

$$\int f(x)dx$$

He integral is called an indefinite integral, which represents a class of functions (the anti-derivative) whose derivative is the integrand. The fundamental theorem ofcalculus relates the evaluation of definite integrals to indefinite integrals. There are

several extensions of the notation for integrals to encompass integration on unbounded domains and/or in multiple dimensions (see later sections of this article).

In advanced settings, it is not uncommon to leave out *dx* when only the simple Riemann integral is being used, or the exact type of integral is immaterial. For instance, one mightwrite

$$\int_{a}^{b} (c1\, f + c2g) = c1 \int_{a}^{b} f + c2 \int_{a}^{b} g$$

to express the linearity of the integral, a property shared by the Riemann integral and allgeneralizations thereof.

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Interpretations

Integrals appear in many practical situations. For instance, from the length, width and depth of a swimming pool which is rectangular with a flat bottom, one can determine thevolume of water it can contain, the area of its surface, and the length of its edge. But if itis oval with a rounded bottom, integrals are required to find exact and rigorous values for these quantities. In each case, one may divide the sought quantity into infinitely many infinitesimal pieces, then sum the pieces to achieve an accurate approximation.

For example, to find the area of the region bounded by the graph of the function *f*(*x*) = between *x* = 0 and *x* = 1, one can cross the interval in five steps (0, 1/5, 2/5... 1),then fill a rectangle using the right end height of each piece (thus √0 , √1/5 , √2/5 …..√1 and sum their areas to get an approximation of

$$\sqrt{\frac{1}{5}}\left(\frac{1}{5}-0\right)+\sqrt{\frac{2}{5}}\left(\frac{2}{5}-\frac{1}{5}\right)+\cdots+\sqrt{\frac{5}{5}}\left(\frac{5}{5}-\frac{4}{5}\right)\approx 0.7497$$

Which is larger than the exact value. Alternatively, when replacing these subintervals byones with the left end height of each piece, the approximation one gets is too low: with twelve such subintervals the approximated area is only 0.6203. However, when the number of pieces increases to infinity, it will reach a limit which is the exact value of the area sought (in this case, 2/3). One writes

$$\int_0^1 \sqrt{x}\, dx = \frac{2}{3}$$

Which means 2/3 is the result of a weighted sum of function values, $\sqrt{x}$ multiplied by the infinitesimal step widths, denoted by dx, on the interval [0, 1].

Overview

Applications of the Integral

Calculus is an interesting subject to analyze from a learning and teaching perspective, given that it has so many facets. This study will focus on integral calculus, as it is usually taught in a standard second university calculus course, supposing the continuation of a standard sequence which means that differentiation, an introduction to integration through the ideas of anti-derivative and Riemann sums, as well as the Fundamental Theorem of Calculus, were included in the first course, calculus I. Approximately the first half or more of the second calculus course typically concentrates on applications and techniques that use the integral as an operational tool to calculate areas, volumes of solids of revolution, surface areas of surfaces of revolution, arc length, and physical applications, as well as instructing the students in multiple procedures that rely strongly on their background in algebra and trigonometry. When dealing with the importance and nature of background, the relevance of mathematical fluency will be emphasized. The very nature of the applications taught in the calculus II course, such as generating volumes and discerning between the disk and shell methods, call upon geometric intuition; in the same vein, a certain degree of analytic sophistication, plus strong algebra and trigonometric skills, are a requisite to be able to understand and operate with integration techniques. Added to this, as the student has already been exposed, in a first course, to the Fundamental - 4 - Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Theorem of Calculus and Riemann sums, it is expected that the processes mentioned above will be situated in a global context, in which the integral itself is understood outside of its varied interpretations, uses and manipulations, as an object that represents accumulation (whether of area, distances, volumes, work). The idea of the dual nature of symbolism in mathematics, as representing processes and concepts (Tall, 1994), is a natural reference frame. Language is an aspect of utmost importance as well, given that it is in the calculus context where the student sees "a new type of math from what we've learned all our lives" (Frid, 1994, p. 80). The symbolic language used in calculus, together with the new or deeper meaning given to familiar terms (Pimm, 1987), will be analyzed in this study, as related to the concept of integration. It is also of interest to test and see if the way that the integral and the integration process is presented in the first course, through Riemann sums and as representing the area under the curve, can convert itself into an obstacle in terms of the flexibility and capacity of generalization that the student needs to be successful in the study of integral calculus. It has been questioned (Czarnocha & Prabhu, 2001; Cordero, 1989) if the introduction of the definite integral as synonymous with the area under the curve could be counter-productive when attempting to expand this important, but particular, interpretation of the definite integral.

Summary

The elements of the theoretical framework provide the language and concepts to analyze mathematical fluency (efficiency, accuracy and flexibility) as measured by the four parameters of foreign language learning: reading comprehension, listening comprehension, speaking and writing. In particular, the concept of "precepts" helps analyze and pinpoint difficulties that the unsuccessful student might show, as the integral symbol is a prototype of the symbol that simultaneously represents a process (integration) and a concept (accumulation). The four-stage model of mathematical learning provides a categorization which is very useful to understand the learning of new concepts, and gives precise terms that can be used to classify students. Mental models, schemas and strategies are concepts that can also be detected by the parameters of mathematical fluency, and can help shed light on students' performance. The use of extra mathematical and structural metaphors, some of which are helpful and others which are misleading or plainly wrong, are detected in this study. Cognitive obstacles are also studied in the present work. The concept of mathematical fluency considers and incorporates previous fluency research, but complements and expands it.

Mathematical fluency is a broader concept that borrows from foreign language learning by incorporating the four parameters of reading comprehension, listening comprehension, speaking and writing, but is situated within the domain of mathematics and mathematics -49- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Learning. It should be made clear that the parameters of mathematical fluency are not a means of promoting or developing fluency, but a measurement technique. In the particular context of this study, speaking, for example, is not considered as a means of developing fluency (as in other research and pedagogical contexts), but as an indicator of mathematical fluency itself. In the present study, the precept that amalgamates the process and concept represented by the integration symbol $\int$ is identified through the parameters of mathematical fluency. The students who evidence maturation in terms of this mental structure (the precept itself), will be seen to have demonstrated higher levels of performance and deeper conceptual comprehension. Classification based upon the four stage model of mathematical learning is made, and it will be seen that this classification is concept- specific; that is, a particular student may be at the integration stage in, say, the use of the disk and shell methods to set up integrals that represent volumes of revolution, while being at the analysis stage when working with functions or trigonometric relationships. The study of students' mental models, identified through the use of the four parameters, leads to the analysis of schemas. It will be seen how important schemas are when students are introduced to new concepts so heavily related to previous geometric and algebraic knowledge. When realizing qualitative research in mathematics education, and especially when using interview techniques, students' use of metaphors can be systematized. This can lead to the detection of patterns of (is age, as well as to the detection of individual usage of metaphors; such usage can be either helpful or misleading. There are even certain metaphors, both -50- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Extra-mathematical and structural, which can play a temporary role in understanding, and must be discarded or modified when dealing with future material. This ties right in to the subject of cognitive obstacles, especially the ones defined as didactical, and which usually have been introduced by an educator as a means of facilitating the learning of a particular concept, but which, on the long run, should be discarded when dealing with future concepts. Often these metaphors or "rules" are carried over into mathematical realms in which they are not useful, or are even false, and it is not clear to the student why they work in one context, and not in the new one. The detection of mathematical fluency as defined in this chapter, and how it relates to students' success with applications, as well as to their comprehension of the underlying notions of calculus, is the goal of the present study. The theoretical framework presented in this chapter is intended to set the stage for the actual research. Although Tall and Gray's definition of precepts, as mental structures that amalgamate the processes and concepts inherent in the same mathematical symbol, together with Kinsley's four stage model of mathematical learning, Primm's extra mathematical and structural metaphors, Brousseau's cognitive obstacles, and the general work done on mental models, schemas and strategies, are the basic providers of the language and concepts used to describe mathematical fluency as detected by the parameters of foreign language learning, other concepts such as abstraction and concept image will also be mentioned in the chapters on results and conclusions. For that reason, this chapter's intent was to offer all the elements needed to frame the present study; also included are some elements -51 - Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. not explicitly called upon when presenting the results and conclusions, but whose presentation could help in the understanding of the foundations upon which the present study has been built.

LEARNING AND CALCULUS

There has been a lot of activity in terms of research and reflection on calculus teaching in the past twenty years, not only in the United States, with the "Calculus Reform", but in other countries as well. The "Calculus Reform" in the United States has produced many studies, the majority dealing with comparative questions between the reform and standard calculus courses. The majority of these studies do not shed much light on the research questions of this study. On the other hand, studies dealing with conceptual and cognitive issues in calculus learning are not so abundant, the majority of them addressing the concepts of limit and derivative, more than issues related to the integral. The research on conceptual questions related to the mathematical object "integral" takes place in the setting of the standard second calculus course, which assumes that the student has studied the Fundamental Theorem of Calculus. That is why this literature review will begin with some research that has been done on students' understanding of the Fundamental Theorem of Calculus. -53- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. The Fundamental Theorem of Calculus Several authors (Kinsley, 2000; Tucker, 1995; Carlson, Persson, Smith, 2003) have commented that, instead of unleashing an important conceptual insight that links the two basic aspects of calculus, students find, when presented with the Fundamental Theorem of Calculus, that "the amazing connection between differentiation and integration is anti-climactic, at best." The study on the Fundamental Theorem of Calculus most related to the present study is by P. Thompson (1994) in which, through a teaching experiment in a course, especially devised, on computers in teaching mathematics, and geared towards a relatively "sophisticated" group of students (7 senior mathematics majors, 10 masters students in secondary mathematics education, 1 masters student in applied mathematics and 1 senior elementary education major, having completed at least 3 semesters of calculus), he studied students' insights into the Fundamental Theorem of Calculus. The analysis was divided in three parts: "Students' images as expressed during the teaching experiment and their contribution to students' difficulties, issues of notation, and implications of the ... teaching experiment for standard approaches to the Fundamental Theorem and introductory calculus in general." (p. 268). The issue of notation will be discussed below, in the subsection on notation, as the author himself commented that "Their orientation toward notational opacity, while having nothing to do with conceptual difficulties with the Fundamental Theorem of Calculus as such, certainly contributed to their -54- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Not having grappled with key connections." (p. 270). This last aspect, the "key connections", was a very important finding of this teaching experiment. In the transcriptions of the students' participation in class, as well as the results of the follow up assessment which consisted of "...four items to clarify possible sources of difficulty - two items on interpreting a difference quotient and two items on Riemann sums as function", the lack of connections was evident in the majority of the students in every question. In particular, it was evident that the main idea of the Riemann sum as a function was not understood. On one question, a graph was given as a function q(x) and the students were asked to sketch a graph of
Only 8 out of the 17 students sketched appropriate graphs.

Problem Solving

Problem solving may be one of the most important indicators of transference of knowledge and techniques (strategies), within the same domain and between domains. The literature on problem solving and integral calculus is, almost exclusively, related to pedagogical approaches, in particular reform versus standard, but not analysis of mental models, metacognition, abstraction, transference, etc. However, there are some studies related to calculus problem solving in general. Cifarelli (1993) realized a study in which he wanted to follow the representation processes in the problem solving situation. The subjects were from freshmen calculus courses at the University of San Diego. The author mentioned that "...the cognitive studies that have been undertaken have seldom focused on the ways that learners

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actively modify their problem representations when they encounter problematic situations." The idea of this study was to see how problem posing itself is implicit in the process of problem solving. Cifarelli also mentions that "Research suggests that the success of capable problem solvers may be due ... to their ability to construct appropriate problem representations in problem solving situations to use as aids for understanding the information and relationships of the situation at hand." This is an approach that differs from that used in studies cited by Cifarelli, where "a solver's ability to recognize similarity across tasks that embody similar 'problem structures' is taken as evidence that the solver has developed an appropriate problem - 64- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. Representation." That is why this study in particular was designed to "...acquire an understanding of the processes used by learners to construct and/or modify problem representations in problem solving situations." (pp. 3, 4). The study was realized through interviews, in which it was possible to observe the subjects experiencing "dilemmas", which, if things went well, provided them with the opportunity to solve these dilemmas and, in the process, further their conceptual knowledge. Of course, by "solving a dilemma", the student was making progress in the problem solving process by abstraction in a domain specific context. At the same time, the chaining process occurs, because once the problem solver modifies the problem representation, the hierarchies of mental objects upon which he is working, as well as the meanings (correct or not) are transformed (for example, once the function for an optimization problem is created, it becomes the "object"; it also turns from "signifier" into "signified" as it is operated upon). Another study which deals with calculus and problem solving is the article "Even Good Calculus Students Can't Solve Non routine Problems" (Mason, Selden & Selden, 1994). In this study, part of a series that the authors realized, a group of A and B calculus students who had taken first semester calculus the semester before the study, were given a set of non-routine problems to solve, as well as a set of routine problems that tested their general knowledge and memory of what they had seen in their calculus class. They were compared with a group of C students who had been given the same routine and non-routine problems in a previous study and who had been unable to solve problems for which they had not been taught a method of solution, even though they had sufficient knowledge -65- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission. To solve the problems. The A and B students did not fare much better, as "two thirds of the students could not solve a single problem correctly.", It was interesting to see that, according to the authors, students who monitored their work did somewhat better; even though it wasn't pointed out in the article, this is very related to the findings of Cifarelli, in terms of the importance of representation and mental models within the solving process of a particular problem, not just in terms of generalization across problems. This also gives evidence of the presence and.importance of chaining. It is also a consensus that the accumulation of more factual knowledge will not necessarily lead to this non routine problem solving ability. Another important aspect to take into account from the results of this study (and related to the study by Arslan, Chaachoua and Laborde mentioned in the subsection on notation), is that the students tended to look for the solution to calculus problems in arithmetic and algebraic techniques more than in the calculus knowledge that they knew they were being tested on. The authors raise the question of whether it takes time to get comfortable with a new subject, and perhaps the students at the end of the two year sequence (calculus and differential equations) would be more inclined to make use of calculus.

Check the formula of integration.
1. Basic integration formulas
2. Integral of special functions
3. integral by partial functions
4. Integration by part
5. Other special integrals
6. Area as a sum
7. Properties of definite integration
Integration of trigonometric functions, properties of definite integration are all mentioned here.

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BASIC FORMULA

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$. Particularly, $\int dx = x + c$
2. $\int \cos x\,dx = \sin x + c$
3. $\int \sin x\,dx = -\cos x + c$
4. $\int sec^2\ x\,dx = \tan x + c$
5. $\int cosec^2\ dx$ = -cot x + c
6. $\int \sec x \tan x\,dx = \sec x + c$
7. $\int cosec\ x \cot x\ dx = -cosec\ x + c$
8. $\int \frac{dx}{\sqrt{1-x^2}} = sin^{-1} x + c$
9. $\int \frac{dx}{\sqrt{1-x^2}} = -\ cos^{-1} x + c$
10. $\int \frac{dx}{\sqrt{1-x^2}} = tan^{-1} x + c$
11. $\int \frac{dx}{\sqrt{1-x^2}} = cot^{-1} x + c$
12. $\int e^x\,dx = e^x + c$
13. $\int a^x\ dx = \frac{a^x}{lo} + c$
14. $\int \frac{dx}{x\sqrt{x^2-1}} = sec^{-1} x + c$
15. $\int \frac{dx}{x\sqrt{x^2-1}} = -\ cosec^{-1} x + c$
16. $\int \frac{1}{x}\,dx = \log|x| + c$
17. $\int \tan x\ dx = \log|\sec x| + c$
18. $\int \cot x\ dx = \log|\sin x| + c$
19. $\int \sec x\ dx = \log|\sec x + \tan x| + c$
20. $\int \operatorname{cosec} x\ dx = \log|\operatorname{cosec} x - \cot x| + c$

Integrals of some special function s

1. $\int dx/(x^2 - a^2) = 1/2a\ \log|(x - a)/(x + a)| + c$
2. $\int dx/(a^2 - x^2) = 1/2a\ \log|(a + x)/(a - x)| + c$
3. $\int dx / (x^2 + a^2) = 1/a\ \tan^{(-1)} x / a + c$
4. $\int dx / \sqrt{(x^2 - a^2)} = \log|"x" + \sqrt{(x^2 - a^2)}| + C$
5. 1.$\int dx / \sqrt{(a^2 - x^2)} = \sin\text{-}1\ x / a + c$
6. $\int dx / \sqrt{(x^2 + a^2)} = \log|"x" + \sqrt{(x^2 + a^2)}| + C$

Integrals by partial fractions

1. $(px + q) / ((x - a)(x - b)) = A/(x - a) + B/(x - b)$
2. $(px + q) / (x - a)^2 = A/(x - a) + B/(x - a)^2$
3. $(px^2 + qx + r) / (x - a)(x - b)(x - c) = A/(x - a) + B/(x - b) + C/(x - c)$
4. $(px^2 + qx + r) / ((x - a)^2 (x - b)) = A/(x - a) + B/(x - a)^2 + C/(x - b)$
5. $(px^2 + qx + r) / (x - a)(x^2 + bx + c) = A/(x - a) + (Bx + C)/(x^2 + bx + c)$

   Where $x^2 + bx + c$ can not be factorised further.

Integration by parts

1. $\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int (f'(x) \int g(x)\, dx)\, dx$

To decide first function. We use

I → Inverse (Example $\sin^{(-1)} x$)

L → Log (Example log x)

A → Algebra (Example x^2, x^3)

T → Trigonometry (Example $\sin^2 x$)

E → Exponential (Example e^x)

2. $\int ex\,[f(x) + f'(x)]\, dx = \int ex\, f(x)\, dx + C$

Other Special Integrals

1. $\int \sqrt{(x^2 - a^2)}\, dx = x/2 \sqrt{(x^2 - a^2)} - a^2/2 \log|x + \sqrt{(x^2 - a^2)}| + C$
2. $\int \sqrt{(x^2 + a^2)}\, dx = x/2 \sqrt{(x^2 + a^2)} + a^2/2 \log|x + \sqrt{(x^2 + a^2)}| + C$
3. $\int \sqrt{(a^2 - x^2)}\, dx = x/2 \sqrt{(a^2 - x^2)} + a^2/2 \sin^{1} x/a + C$

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Integral of the form ∫ (px+q) √(ax ^{2} + bx + c) dx

We solve this using a specific method.

1. First we write

   px + q = A (d(√(ax ^{2} + bx + c))/dx) + B

2. Then we find A and B

3. Our equation becomes two seperate identities and then we solve.

Integration: An Inverse Process of Differentiation

We are given a function's derivative and asked to discover its primitive or the originalfunction. Anti-differentiation or integration is the term for this type of process. When given a function's derivative, determining the original function is integration.

The integrals and derivatives are opposites of each other. Take the function f(x)=sin x. F(x) = cos x is the derivative of f(x). The function cos x is known as the derived functionof sin x. Sin x is the anti-derivative of cos x and vice versa.

Integration by Parts

Integration by parts is a technique or method for combining the output of two or more functions. f(x) and g(x), the two functions to be integrated, are of the form f(x).g (x). As aresult, it's referred to as a product rule of integration.

We choose f(x) because its derivative formula exists, while we select g(x) due to theexistence of an integral for it.

∫f(x).g(x).dx=f(x)∫g(x).dx − ∫(f′(x)∫g(x).dx).dx + C

Integration by Parts Applications

We can use the formula derived above for functions or expressions with no integration formulas. Here, we will incorporate the parts-integration formula and derive the integral. There are no integral answers for logarithmic and inverse trigonometric functions. Let ustry to solve and find the log x and tan-1x integration.

Integration of Logarithmic Function

∫ logx.dx = ∫ logx.1.dx

= logx. ∫1.dx - ∫ ((log x)'.∫ 1.dx).dx

=logx.x -∫ (1/x .x).dx

=log x - ∫ 1.dx

=x log x - x + C

Integration of Inverse Trigonometric Function

∫ tan-1x.dx = ∫tan-1x.1.dx

= tan-1x.∫1.dx - ∫((tan-1x)'.∫ 1.dx).dx

= tan-1x. x - ∫(1/(1 + x2).x).dx

= x. tan-1x - ∫ 2x/(2(1 + x2)).dx

= x. tan-1x - ½.log(1 + x2) + C

II. CONCLUSION

This article summaries the topic of integration by parts. It also explains how to derivethe formula for integrations by parts.

Finding the region's area under the curve is known as integration. One can accomplishintegration by drawing as many little rectangles as possible to cover the area and adding them together. The sum of all quantities approaches a limit equal to the integration of some special functions under the curve.

Integration is a mathematical technique to calculate the area under a curve. There are multiple methods for integration, of which we explore integration by parts in this article. Here, we derive its integration by parts formula and also understand its applications.

With integration and differentiation as mathematical techniques, I have been able to reach the solution of the different methods used. Each of the methods brings a simple and clear way thus can be used for practical data. Furthermore, with tables and figures have been able and compare the actual data with the predicted data. The analyzing of the different countries that are less developed has given an overview of how overpopulation will cause severe problems for example instability, showing the country's development, the viscous poverty circle, pollution and environmental degradation. Using the techniques above I have been able to predict population sizes with and without family planning. For example I analyzed the family planning policy in china and evaluated how it has advantageously helped china in its development and poverty reduction. Therefore, it will be true if I say that other developing and underdeveloped countries deserve to emphasize on the implementation of the family planning rule in order to reduce the negative impacts of massive population that has affected our world. This project can therefore be viewed as a scientific effort to measure and manage population growth in the current world setup. It can also help struggling or developing nations adopt some of the method displayed above to improve their social and economic status.

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