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On Models of Time Series

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Abstract*: Recently a number of time series models have been constructed and studied for real valued observations using different marginal distributions by several researchers. Many naturally occurring time series observations show a tendency to follow asymmetric and heavy tailed distributions. A review of literature on some heavy tailed models of time series is done in this paper.*

Keywords: Asymmetric Geometric Linnik Distribution**,** Autoregressive model, Geometric Linnik Distribution, Tailed Generalized Geometric Linnik Distribution, Time Series

I. INTRODUCTION

A time series is a series of observations made sequentially in time. Main objective of time series analysis is to reveal the probability law that governs the observed time series. To describe the association between two values on the same series at different times we use the auto correlation function

$$\rho_k = \frac{E\left[(X_t - \mu_t)(X_{t+k} - \mu_{t+k})\right]}{\sqrt{Var\left(X_t\right)Var\left(X_{t+k}\right)}} \qquad (1.\ 1)$$

where $\mu_{t+i} = E\left(X_{t+i}\right), i = 0,1,2,\ldots$.

A time series can be viewed as a realization of a stochastic process. A class of time series we encounter in practical situations is the stationary series. If for any set of time t_1, t_2, …, t_n and at times t_1 +h, t_2+h, …, t_n +h, the joint distribution of $\left(X_{t_1}, X_{t_2}, \ldots, X_{t_n}\right)$ and $\left(X_{t_1+h}, X_{t_2+h}, \ldots, X_{t_n+h}\right)$ are the same for all $h > 0$ and for every n, then the process $\{X_t\}$ is said to be strictly stationary. A time series $\{X_t\}$ is said to be weakly stationary if (i) $E({X_t}^2) < \infty$, (ii) $E(X_t) = \mu$ for all t and (iii) $Cov\,(X_t, X_{t+s})$ depends only on the length of the interval s. Note that $\{X_t\}$ is weakly stationary if it is strictly stationary with finite second moments.

The most popular class of linear time series models consists of autoregressive moving average (ARMA) models, purely autoregressive (AR) and purely moving average (MA) models. For modeling non-stationary data, autoregressive integrated moving average (ARIMA) models are used.

An autoregressive model of order $p \geq 1$, abbreviated as AR(p), is defined as

$$X_n = \rho_1 X_{n-1} + \rho_2 X_{n-2} + \ldots + \rho_p X_{n-p} + \varepsilon_n \qquad (1.2)$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables and $\rho_1, \rho_2, \ldots$ are constants. Note that (1.2) represents the current value of the process X_n through its immediate p past values

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$X_{n-1}, X_{n-2}, ..., X_{n-p}$ and a random shock ε_n. The simplest form of an autoregressive model is AR (1) and is given by

$$X_n = \rho X_{n-1} + \varepsilon_n. \qquad (1.3)$$

Let $\phi_{X_n}(t)$ denotes the characteristic function of X_n and $\phi_{\varepsilon_n}(t)$, that of ε_n. Then (1.3) in terms of characteristic function becomes $\phi_{\varepsilon_n}(t) = \frac{\phi_{X_n}(t)}{\phi_{X_{n-1}}(\rho t)}$. (1.4)

That is, the innovation process exists if and only if (1.4) is a characteristic function for every ρ, $|\rho| \le 1$. In (1.4) the case of stationary process $\{X_n\}$, $\phi_{\varepsilon_n}(t)$ exists for every ρ, $0 < \rho \le 1$ if and only if the distribution of X_n is in class L or self decomposable. The autocorrelation function of the process (1.3) is $\rho(k) = \rho^k$.

A moving average model of order $q \ge 1$, denoted by MA(q), is given by

$$X_n = \delta_1 \varepsilon_{n-1} + \delta_2 \varepsilon_{n-2} + ... + \delta_q \varepsilon_{n-q} + \varepsilon_n \qquad (1.5)$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables and $\delta_1, \delta_2, ..., \delta_q$ are constants. Here the current value of X_n is linearly dependent on the q previous values of ε_n's. For q=1, (1.5) reduces to the MA(1) model given by

$$X_n = \delta_1 \varepsilon_{n-1} + \varepsilon_n. \qquad (1.6)$$

Combining AR and MA models, the general linear time series model, namely autoregressive moving average model, denoted by ARMA (p,q) has the form

$$X_n - \rho_1 X_{n-1} - \rho_2 X_{n-2} - ... - \rho_p X_{n-p} = \varepsilon_n + \delta_1 \varepsilon_{n-1} + \delta_2 \varepsilon_{n-2} + ... + \delta_q \varepsilon_{n-q} \qquad (1.7)$$

where $\rho_1, \rho_2, ..., \rho_p$; $\delta_1, \delta_2, ..., \delta_q$ are constants and $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables.

II. SOME NON GAUSSIAN AUTOREGRESSIVE MODELS

Even though Gaussian models have dominated in the development of time series modeling, autoregressive processes with non Gaussian marginal distributions are a fast growing area of investigation due to the applications of the same .A number of non-Gaussian autoregressive models have been introduced by various researchers.(see [2]).

The study of non-Gaussian autoregressive models began with the pioneering work of [1]. They have considered an AR(1) model with exponential (μ) marginal distribution. The model is given by $X_0 = \varepsilon_1$

and for $n = 1, 2, ...$ $X_n = \rho X_{n-1} + \begin{cases} 0 & w.p. \quad \rho \\ \varepsilon_n & w.p. \quad (1-\rho) \end{cases}$ ……(2.1) and $w.p.$ stands for with probability, $0 \le \rho \le 1$ and $\{\varepsilon_n\}$ is a sequence of independent and identically distributed exponential random variables.

[3 developed time series models with generalized geometric Linnik marginal distribution. .

DEFINITION 2.1

A random variable X on R is said to have geometric Linnik distribution and write $X \underline{\underline{d}} GL(\alpha, \lambda)$ if its characteristic function $\phi(t)$ is

$$\phi(t) = \frac{1}{1+\ln(1+\lambda|t|^{\alpha})}, t \in R, 0 < \alpha \le 2, \lambda > 0. \qquad (2.2)$$

DEFINITION 2.2

A random variable X on R is said to have type I generalized geometric Linnik distribution and write $X \underline{\underline{d}} GeGL_1(\alpha, \lambda, p)$ if it has the characteristic function

$$\phi(t) = \frac{1}{1+p\ln(1+\lambda|t|^{\alpha})}, \; 0 < \alpha \le 2, \; p > 0, \; \lambda > 0. \qquad (2.3)$$

DEFINITION 2.3

A random variable X on R has type II Generalized Geometric Linnik distribution and writes $X \underline{\underline{d}} Ge\, GL_2(\alpha, \lambda, \tau)$, if it has the characteristic function

$$\phi(t) = \left[\frac{1}{1+ln(1+\lambda|t|^{\alpha})}\right]^{\tau}, t \in R, 0 < \alpha \le 2, \lambda, \tau > 0. \qquad (2.4)$$

Note that when $\tau = 1$, type II Generalized Geometric Linnik distribution reduces to geometric Linnik distribution.

THEOREM 2.1

Let $\{X_{n,} n \ge 1 \}$ be defined as

$$X_n = \begin{cases} \varepsilon_n & w.p. \quad p \\ X_{n-1} + \varepsilon_n & w.p. \quad (1-p) \end{cases} \qquad (2.5)$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables. A necessary and sufficient condition that $\{X_n\}$ is strictly stationary Markov process with $GL(\alpha, \lambda)$ marginals is that $\{\varepsilon_n\}$ are distributed as $GeGL_1(\alpha, \lambda, \rho)$.

For proof see [3]

[4] developed time series models with tailed generalized geometric Linnik distribution as marginals.

Definition 2.4

A random variable U is said to have tailed type I generalized geometric Linnik distribution $TGeGL_1(\alpha,\lambda,\tau,\theta)$ if it has the characteristic function

$$\phi_U(t)=\frac{1+\tau\theta\ln\left(1+\lambda|t|^{\alpha}\right)}{1+\tau\ln\left(1+\lambda|t|^{\alpha}\right)}. \qquad (2.6)$$

DEFINITION 2.5

A random variable X is said to have tailed type II generalized geometric Linnik distribution and write $X \underline{\underline{d}}\, TGeGL_2(\alpha,\lambda,\tau,\theta)$ distribution if it has the characteristic function

$$\phi_X(t)=\frac{\theta\left[1+\ln\left(1+\lambda|t|^{\alpha}\right)\right]^{\tau}+(1-\theta)}{\left[1+\ln\left(1+\lambda|t|^{\alpha}\right)\right]^{\tau}},0<\alpha\le 2,0<\theta<1,\ \lambda,\tau>0. \qquad (2.7)$$

[4] developed stationary AR(1) model with tailed type I generalized Linnik process($\mathrm{TGeGL_1}$ distribution as marginal) as follows.

Let $X_0 \underline{\underline{d}}\, TGeGL_1(\alpha,\lambda,\tau,\theta)$ and for $n=1,2,\ldots$

$$X_n=\begin{cases}\varepsilon_n & w.p. \quad p\\ X_{n-1}+\varepsilon_n & w.p. \quad 1-p\end{cases} \qquad (2.8)$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed $TGeGL_1\left(\alpha,\lambda,\tau c,\frac{\tau\theta}{c}\right)$ random variables where $c=p+(1-p)\theta.$

REMARK

If $\{\varepsilon_n\}$ is a sequence of independent and identically distributed $TGeGL_1\left(\alpha,\lambda,\tau c,\frac{\tau\theta}{c}\right)$, where $c=p+(1-p)\theta$ then (2.8) is asymptotically stationary with $TGeGL_1(\alpha,\lambda,\tau,\theta)$ marginal distribution.

Autoregressive models with type I generalized geometric asymmetric Linnik marginals are developed in [5].

DEFINITION 2.6

A random variable X on R is said to have Type I generalized geometric asymmetric Linnik distribution with parameters $\mu, \sigma, \alpha, \tau$ .and write X has distribution $GeGAL_1(\alpha, \lambda, \mu, \tau)$ if it has

the characteristic function $\psi(t) = \dfrac{1}{1+\tau \ln(1+\lambda|t|^{\alpha} - i\mu t)}, -\infty < \mu < \infty, \lambda, \tau \ge 0, 0 < \alpha \le 2$ (2.9)

When μ=0, (2.9) reduces to type I generalized geometric Linnik distribution.

[5] developed a time series model with $GeGAL_1(\alpha, \lambda, \mu, \tau)$ marginal distribution on the basis of geometric infinite divisibility property of the distribution.

THEOREM 2.2

Let $\{X_n, n \ge 1\}$ be defined as

$$X_n = \begin{cases} \varepsilon_n & w.p. \quad \theta \\ X_{n-1} + \varepsilon_n & w.p. \quad 1-\theta \end{cases} \tag{2.10}$$

where $0 < \theta \le 1$ and $\{\varepsilon_n\}$ is a sequence of independent and identically distributed random variables. A necessary and sufficient condition that $\{X_n\}$ is a stationary process with $GeGAL_1(\alpha, \lambda, \mu, \tau)$ marginal is that $\{\varepsilon_n\}$ is distributed as $GeGAL_1(\alpha, \lambda, \mu, \theta\tau)$.

For proof see [5].

The modeling and interpretation of time series data plays a significant role in every field of modern research.The time series models discussed above can be used for modeling interest rates,currency exchange rates,stock price returns,speech waves etc.

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