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A Class of Multivalent Functions Associated with. r^{th} Differential Operator.

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Abstract: *This paper is concern with multivalent functions. We derived the new r^{th} differential operator. Associated with this operator new class of q-valent functions is studied. Sufficient condition for this class has been obtained.*

Keywords: q-valent, multivalent, regular, operator

I. INTRODUCTION

LetR (q) be class of all regular and q-valent functionsin the form

$$f(z)= z^q +\sum_{k=1}^{\infty} h_k z^{k+q}, \qquad (q \in \mathbb{N}) \qquad (1.1)$$

on open unit disc D = {z: $|z| < 1$}.

Goodman [6] and Chaughule[5] have studied necessary and sufficient condition and geometric properties for various subclasses of R(q).These classes includestarlike functions of finite order, close to convex, multivalently convex, etc.

II. MAIN RESULTS

This section is started with new class V (q, r). The class V (q, r) is associated with r^{th} differential operator.[8]. It is expressed as given below:

Definition2.1. Ther^{th} order differential operator [38] for the function f in R (q) is denoted by Q^r. It is given as:

$$Q^r f(z) = \frac{q!}{(q-r)!} z^{q-r} + \sum_{k=q+1}^{\infty} \frac{k!}{(k-r)!} h_k z^{k-r} \qquad (1.2)$$

Where, (q>r, q$\in \mathbb{N}$, r $\in \mathbb{N}_0$)

Definition 2.2. A function in the form 1.1 in R (q) is said to be in the class V (q, r) if it satisfies inequality

$$\left|1+ \frac{zQ^{r+2} f(z)}{Q^{r+1} f(z)} -(q-r)\right| < q-r-1 \qquad (1.3)$$

Where $z \in D, q \in \mathbb{N}, q > r+1$

Further, we find the sufficient condition for this class V (q, r)

Theorem 2.1 If the function f (z) ∈ R (q) satisfies the condition

$$\left| \frac{1+ z\left(\frac{Q^{r+3} f(z)}{Q^{r+2} f(z)} - 1 + \frac{zQ^{r+2} f(z)}{Q^{r+1} f(z)}\right)}{z\left(\frac{Q^{r+2} f(z)}{Q^{r+1} f(z)} - \frac{Q^{r+3} f(z)}{Q^{r+2} f(z)}\right)} \right| < 1. \qquad (1.4)$$

Where $z \in D, q \in \mathbb{N}, q > r+1$

Then f ∈V (q, r).

Proof. Given that $f(z)= z^q +\sum_{k=1}^{\infty} h_k z^{k+q}$ having condition (1.4).

Define $T(z) = \frac{1}{q-r-1}\left(\frac{zQ^{r+2} f(z)}{Q^{r+1} f(z)}\right) - 1$.

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Hence, $\frac{zQ^{r+2}f(z)}{Q^{r+1}f(z)} = (q-r-1)(1+T(z))$

Clearly T (0) =0

With some simplification, we get

$\frac{zT'(z)}{1+T(z)} = \frac{zQ^{r+3}f(z)}{Q^{r+2}f(z)} +1 - \frac{zQ^{r+2}f(z)}{Q^{r+1}f(z)}$

.

$\Rightarrow \frac{zQ^{r+3}f(z)}{Q^{r+2}f(z)} +1= \frac{zT'(z)}{1+T(z)} + (q-r-1)(1+t(z))$

$$\Rightarrow \left| \frac{1+\ z\left(\frac{Q^{r+3}f(z)}{Q^{r+2}f(z)} + \frac{zQ^{r+2}f(z)}{Q^{r+1}f(z)}\right)}{z\left(\frac{Q^{r+2}f(z)}{Q^{r+1}f(z)} - \frac{Q^{r+3}f(z)}{Q^{r+2}f(z)}\right)} \right| = \left| \frac{\frac{zT'(z)}{1+T(z)}}{1-\frac{zT'(z)}{1+T(z)}} \right|$$

We now claim that |T (z)| <1, z∈ D. Conversely we assume that |T (z)| ≥1.

Then by Jack's lemma [7] there exist $z_1 \in D$ such that that |T (z_1)| ≥1. and $z_1T'(z1)$=aT (z_1) $for\ a \geq 1$.

Using (1.4)

$$\left| \frac{1+\ z_1\left(\frac{Q^{r+3}f(z_1)}{Q^{r+2}f(z_1)} - 1 + \frac{Q^{r+2}f(z_1)}{Q^{r+1}f(z)}\right)}{z_1\left(\frac{Q^{r+2}f(z_1)}{Q^{r+1}f(z_1)} - \frac{Q^{r+3}f(z_1)}{Q^{r+2}f(z_1)}\right)} \right| = \left| \frac{\frac{z_1T'(z_1)}{1+T(z_1)}}{1-\frac{z_1T'(z_1)}{1+T(z_1)}} \right|$$

$= \left| \frac{\frac{aT'(z_1)}{1+T(z_1)}}{1-\frac{aT'(z_1)}{1+T(z_1)}} \right| = \left| \frac{aT(z_1)}{aT(z_1)-1-T(z_1)} \right| \geq 1.$

This contradicts to (1.4). Hence T (z) < 1. For z in D.

Therefor f∈ $V(q,r)$.

Example 1.1. If f ∈ $R(q)$ satisfying inequality

$\left| \frac{1+\ z\left(\frac{f^3(z)}{f^2(z)} - \frac{f^2f(z)}{f^1(z)}\right)}{z\left(\frac{f^2(z)}{f^1(z)} - \frac{Q^{r+3}f(z)}{Q^{r+2}f(z)}\right)} \right| < 1$

Where $z \in D, r \in \mathbb{N}_0, q > r+1$, q ∈ ℕ,

The f is multivalently convex function in D.

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