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Robust Stabilization of Uncertain Jerk Chaotic Control Systems with Mixed Uncertainties

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Abstract*: In this paper, the problem of robust stabilizability of jerk chaotic control systems with mixed uncertainties is investigated. Combining robust control theory and differential-integral inequalities, a nonlinear controller will be derived and guaranteed to achieve the goal of practical stabilization. Besides, both the convergence radius and the exponential convergence rate can be specified in advance. Finally, some numerical simulation results are supplemented to demonstrate the correctness and effectiveness of the main result*

Keywords: Robust Stabilization, Uncertain systems, Chaotic system, Mixed Uncertainties

I. INTRODUCTION

In recent years, there have been many related studies on chaotic systems; see, for example, [1]-[12] and the references therein. The concept of practical stabilization has been first proposed in [7] and has been proven to be very effective in chaos suppression. Since the controller design of practical stabilization meets the requirements of both the transient response and the steady-state response of the system, it is indeed very effective in the design of controllers for chaotic systems.

As we know, it is not an easy task to find a controller that can simultaneously overcome input nonlinearities and mixed uncertainties and suppress chaotic oscillations. Furthermore, a control system that can achieve any specified convergence radius and any specified exponential convergence rate at the same time has always been a dream goal of control engineers. To be fair, finding a controller that enables a closed-loop control system to achieve high-quality transient response and excellent steady-state response is definitely the dream of most control engineers, and it is also a task that is not easy to achieve.

Motivated by the concept of practical stabilization, this paper intends to design a controller for a class of uncertain chaotic control system with multiple uncertainties, so that the closed-loop system can simultaneously achieve the pre-specified convergence radius and exponential convergence rate. Throughout this paper, some symbols are defined as follows:

$\lvert a\rvert$	the modulus of a complex number a
I	the unit matrix
A^T	the transport of the matrix A
$\lVert x\rVert$	the Euclidean norm of the vector $x \in \Re^n$
$\lambda_{\min}(A)$	the minimum eigenvalue of the matrix A with real eigenvalues
$\sigma(A)$	the spectrum of the matrix A

II. PROBLEM FORMULATION AND MAIN RESULT

In this paper, we explore the well-known second-order uncertain jerk chaotic control systems with mixed uncertainties [2] described as

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$$\ddot{y}(t) = q_1\dot{y}(t) + q_2 y^3(t) + q_3\cos(wt) + \Delta f(y,\dot{y}) + \Delta\phi(u),\ \forall t \ge 0.$$

The state variable expression of the above dynamic system is as follows:

$$\dot{x}_1(t) = x_2(t), \quad \text{(1a)}$$

$$\dot{x}_2(t) = q_1 x_2(t) + q_2 x_1^3(t) + q_3\cos(wt) + \Delta f(x_1(t), x_2(t)) + \Delta\phi(u(t)),\ \forall t \ge 0, \quad \text{(1b)}$$

where $x(t) = [x_1(t) \quad x_2(t)]^T \in \Re^{2\times 1}$ is the state vector, $u(t) \in \Re$ is the input, $\Delta f(x_1(t), x_2(t))$ means the mixed uncertainties (parameter mismatchings and external excitations), and $\Delta\phi(u(t))$ means the unknown input nonlinearity. To ensure the existence of the solutions of (1), we assume that the unknown terms $\Delta f(x_1, x_2)$ and $\Delta\phi(u)$ are all continuous functions. It is worth mentioning that system (1) exhibits chaotic behavior for certain parameter values when there are no uncertain terms (i.e., $\Delta f(x_1, x_2) = \Delta\phi(u) = 0$) [2]. In this paper, we hope to design a controller that can not only overcome input nonlinearities, mixed uncertainties, and chaotic vibrations at the same time, but also achieve the goals of any specified convergence radius and any specified exponential convergence rate.

For the uncertain terms $\Delta f(x_1, x_2)$ and $\Delta\phi(u)$, we make the following assumption:

(A1) There exist continuous function $f(x_1, x_2) \ge 0$ and positive number r_1 such that, for all arguments,

$$|\Delta f(x_1, x_2)| \le f(x_1, x_2),$$

$$u \cdot \Delta\phi(u) \ge r_1 u^2.$$

A precise definition of the practical stabilization is presented below.

Definition 1 [7]: The uncertain system (1) is said to achieve the practical stabilization, provided that, for any $\alpha > 0$ and $\varepsilon > 0$, there exists a control $u := u(\alpha, \varepsilon)$ such that the state trajectory satisfies

$$\|x(t)\| \le \kappa \cdot e^{-\alpha t} + \varepsilon, \quad \forall t \ge 0,$$

for some $\kappa > 0$. In this situation, the positive number ε is called the convergence radius and the positive number α is called the exponential convergence rate. In other words, practical stabilization means that the state of system (1) can converge to the equilibrium point at $x = 0$, with any prespecified convergence radius and exponential convergence rate. There is no doubt that the control system with small convergence radius and large exponential convergence rate has better steady-state response and transient response.

Now we put forward the main result for the practical stabilization of uncertain systems of (1).

Theorem 1. The uncertain systems (1) with (A1) are practical stabilization under the following control

$$u(t) = -r(t)\cdot(p_3 x_1 + p_2 x_2), \quad \text{(2)}$$

$$r(t) := \frac{h^2(t)}{r_1 \cdot h(t) \cdot |p_3 x_1 + p_2 x_2| + \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)}, \quad \text{(3)}$$

$$h(t) := \left|q_1 x_2 + q_2 x_1^3 + q_3\cos(wt) + (\alpha+1)^2 x_1 + 2(\alpha+1)x_2\right| + f(x_1, x_2), \quad \text{(4)}$$

where $P := \begin{bmatrix} p_1 & p_3 \\ p_3 & p_2 \end{bmatrix} > 0$ is the unique solution to the following Lyapunov equation

$$\begin{bmatrix} \alpha & 1 \\ -(\alpha+1)^2 & -\alpha-2 \end{bmatrix}^T P + P\begin{bmatrix} \alpha & 1 \\ -(\alpha+1)^2 & -\alpha-2 \end{bmatrix} = -2I, \quad \text{(5)}$$

with $\varepsilon > 0$ and $\alpha > 0$. In this situation, the guaranteed convergence radius and exponential convergence rate are ε and α, respectively.

Proof. From (1)-(5), the state equation of the entire closed-loop control system can be expressed as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -(\alpha+1)^2 & -2\alpha-2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \left[\Delta\phi + \Delta f + q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right]$$
$$= Ax + B \cdot \left[\Delta\phi + \Delta f + q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right], \forall\, t \ge 0,$$

where $A := \begin{bmatrix} 0 & 1 \\ -(\alpha+1)^2 & -2\alpha-2 \end{bmatrix}$ and $B := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Apparently, one has $\sigma(A) = \{-\alpha-1\}$, which implies $A + \alpha I$ is Hurwitz and the Lyapunov equation of (5) has the unique posive definite solution P. Let

$$V(x(t)) = x^T(t) P x(t). \quad (6)$$

The time derivative of $V(x(t))$ along the trajectories of the system (1) with (2)-(5) can be derived as

$$\dot{V}(x(t)) = x^T \left[A^T P + PA \right] x + 2x^T PB \cdot \left[\Delta\phi + \Delta f + q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right]$$
$$\le x^T \left[-2\alpha P - 2I \right] x + 2x^T PB \cdot \Delta\phi + 2\left| x^T PB \right| \left[\left| q_1 x_2 + q_2 x_1^3 + q_3 \cos(wt) + (\alpha+1)^2 x_1 + (2\alpha+2) x_2 \right| + f \right]$$
$$\le -2\alpha x^T P x + 2x^T PB \cdot \Delta\phi + 2h(t) \cdot \left| x^T PB \right|$$
$$= -2\alpha x^T P x - \left(\frac{2}{r} \right) \cdot (u \cdot \Delta\phi) + 2h \cdot \left| x^T PB \right|$$
$$\le -2\alpha V - \left(\frac{2}{r} \right) \cdot \left(r_1 u^2 \right) + 2h \cdot \left| x^T PB \right|$$
$$= -2\alpha V - 2r_1 \cdot r \cdot \left| x^T PB \right|^2 + 2h \cdot \left| x^T PB \right|$$
$$= -2\alpha V - 2r_1 \cdot r \cdot \left| p_3 x_1 + p_2 x_2 \right|^2 + 2h \cdot \left| p_3 x_1 + p_2 x_2 \right|$$
$$= -2\alpha V - \frac{2r_1 \cdot \left| p_3 x_1 + p_2 x_2 \right|^2 \cdot h^2}{r_1 \cdot h \cdot \left| p_3 x_1 + p_2 x_2 \right| + \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)} + 2h \cdot \left| p_3 x_1 + p_2 x_2 \right|$$
$$= -2\alpha V + \frac{2h \cdot \left| p_3 x_1 + p_2 x_2 \right| \cdot \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)}{r_1 \cdot h \cdot \left| p_3 x_1 + p_2 x_2 \right| + \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)}$$
$$= -2\alpha V + \left(\frac{2}{r_1} \right) \left[\frac{r_1 \cdot h \cdot \left| p_3 x_1 + p_2 x_2 \right| \cdot \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)}{r_1 \cdot h \cdot \left| p_3 x_1 + p_2 x_2 \right| + \alpha \cdot r_1 \cdot \varepsilon^2 \cdot \lambda_{\min}(P)} \right], \quad \forall\, t \ge 0. \quad (7)$$

Combining the inequality

$$x \left(\frac{yz}{y+z} \right) \le xz, \quad \forall\, x > 0, \; y \ge 0, \text{ and } z > 0$$

and (7), we can obtain

$$\dot{V}(x(t)) \le -2\alpha V(x(t)) + 2\alpha \cdot \varepsilon^2 \cdot \lambda_{\min}(P), \quad \forall\, t \ge 0.$$

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It results that

$$e^{2\alpha t}\cdot\dot{V}(x(t))+2\alpha\cdot e^{2\alpha t}\cdot V(x(t))\le 2\alpha\cdot\varepsilon^2\cdot\lambda_{\min}(P)\cdot e^{2\alpha t}$$

$$\Rightarrow\quad \frac{d\left[e^{2\alpha t}\cdot V(x(t))\right]}{dt}\le 2\alpha\cdot\varepsilon^2\cdot\lambda_{\min}(P)\cdot e^{2\alpha t}$$

$$\Rightarrow\quad e^{2\alpha t}\cdot V(x(t))-V(x(0))=\int_0^t\frac{d\left[e^{2\alpha t}\cdot V(x(t))\right]}{dt}dt\le\int_0^t 2\alpha\cdot\varepsilon^2\cdot\lambda_{\min}(P)\cdot e^{2\alpha t}dt$$

$$=\varepsilon^2\cdot\lambda_{\min}(P)\cdot\left(e^{2\alpha t}-1\right),\quad\forall\, t\ge 0.\qquad(8)$$

It can be readily obtained that

$$\lambda_{\min}(P)\cdot\|x(t)\|^2\le V(x(t))\le e^{-2\alpha t}\left[V(x(0))-\varepsilon^2\cdot\lambda_{\min}(P)\right]+\varepsilon^2\cdot\lambda_{\min}(P),\quad\forall\, t\ge 0.$$

in view of (6) and (8). Consequently, we conclude that

$$\|x(t)\|\le\sqrt{e^{-2\alpha t}\cdot\left[\frac{V(x(0))+\varepsilon^2\cdot\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]+\varepsilon^2}$$

$$\le\sqrt{e^{-2\alpha t}\cdot\left[\frac{V(x(0))+\varepsilon^2\cdot\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]}+\sqrt{\varepsilon^2}$$

$$=\sqrt{\left[\frac{V(x(0))+\varepsilon^2\cdot\lambda_{\min}(P)}{\lambda_{\min}(P)}\right]}\cdot e^{-\alpha t}+\varepsilon,\quad\forall\, t\ge 0.$$

Thus the proof is completed. □

Remark 1.

We provide a procedure to find the robust control law stated in Theorem 1.

INPUT: The uncertain jerk chaotic control systems (1), the pre-specified exponential decay rate α, and the pre-specified convergence radius ε.

OUPUT: Robust control of (2).

Step one. Choose $f(x_1,x_2)$ and $r_1(x_1,x_2)$ such that (A1) is satisfied.

Step two. Calculate P, $\lambda_{\min}(P)$, p_2, and p_3, from (5).

Step three. Form $h(t)$ from (4).

Step four. Form $r(t)$ from (3).

Step five. OUPUT $u(t)=-r(t)\cdot(p_3x_1+p_2x_2)$.

III. NUMERICAL EXAMPLE

Consider the following jerk chaotic control systems with mixed uncertainties:

$$\dot{x}_1=x_2,\qquad(9a)$$

$$\dot{x}_2=q_1x_2+q_2x_1^3+q_3\cos(wt)+\Delta f(x_1,x_2)+\Delta\phi(u),\ \forall\, t\ge 0,\qquad(9b)$$

where

$$q_1=-0.06,\ q_2=-1,\ q_3=5,\ w=1,$$

$$\Delta f(x_1,x_2)=\Delta a\cdot x_2^2,\quad \Delta\phi(u)=\Delta b\cdot u+\Delta c\cdot u^5,$$

$$-1\le\Delta a\le 1,\quad 1\le\Delta b\le 5,\quad \Delta c\ge 0.$$

In this example, our goal is to design a feedback control such that the uncertain system (9) the practical stabilization with the exponential convergence rate $\alpha = 2$ and the convergence radius $\varepsilon = 0.1$.

Step one. When we choose $f(x_1, x_2) = x_2^2$ and $r_1 = 1$, obviously condition (A1) will be satisfied.

Step two. From (5), we have

$$P = \begin{bmatrix} 49 & 11 \\ 11 & 3 \end{bmatrix}, \quad \lambda_{\min}(P) = 0.51, \quad p_2 = 3, \quad p_3 = 11.$$

Step three. From (4), it is easy to deduce that

$$h(t) := \left| -0.06x_2 - x_1^3 + 5\cos t + 9x_1 + 6x_2 \right| + x_2^2.$$

Step four. From (3), we have

$$r(t) := \frac{h^2(t)}{h(t) \cdot |11x_1 + 3x_2| + 0.0102}.$$

Step five. The robust controller, from (2), can be obtained as

$$u(t) = -r(t) \cdot (11x_1 + 3x_2). \qquad (10)$$

Therefore, according to Theorem 1, we conclude that system (9) with the control (10) is practically stable, with the exponential convergence rate $\alpha = 2$ and the guaranteed convergence radius $\varepsilon = 0.1$.Typical state trajectories of uncontrolled and controlled systems are shown in Figure 1 and 2, respectively. In addition, the time response of the control signal is shown in Figure 3. It can be seen from the above simulation results that the uncertain dynamic systems (9) combined with the controller (10) can indeed achieve practical stabilization.

IV. CONCLUSION

In this paper, the problem of robust stabilizability of jerk chaotic control systems with mixed uncertainties has been investigated. Combining robust control theory and differential-integral inequalities, a nonlinear controller has been derived and guaranteed to achieve the goal of practical stabilization. Besides, both the convergence radius and the exponential convergence rate can be specified in advance. Finally, some numerical simulation results have been supplemented to illustrate the effectiveness and correctness of the obtained results.

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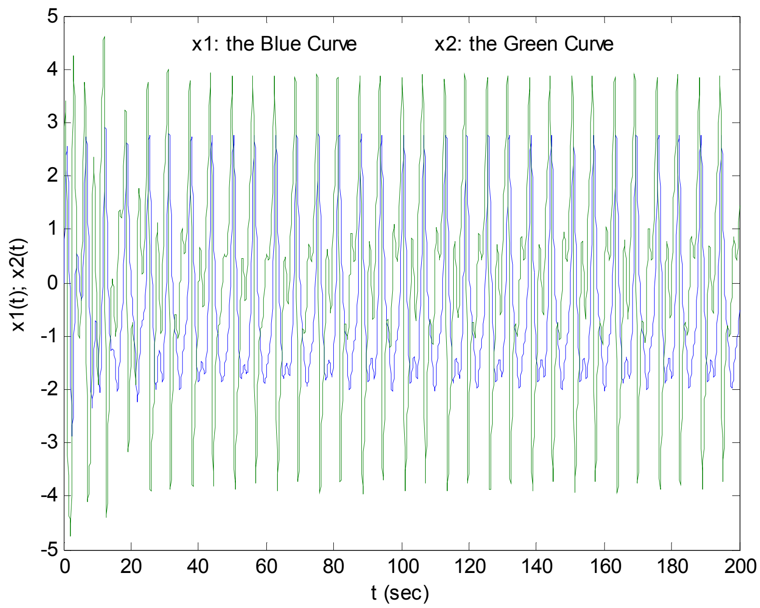

Figure 1: Typical state trajectories of the uncontrolled system of (9).

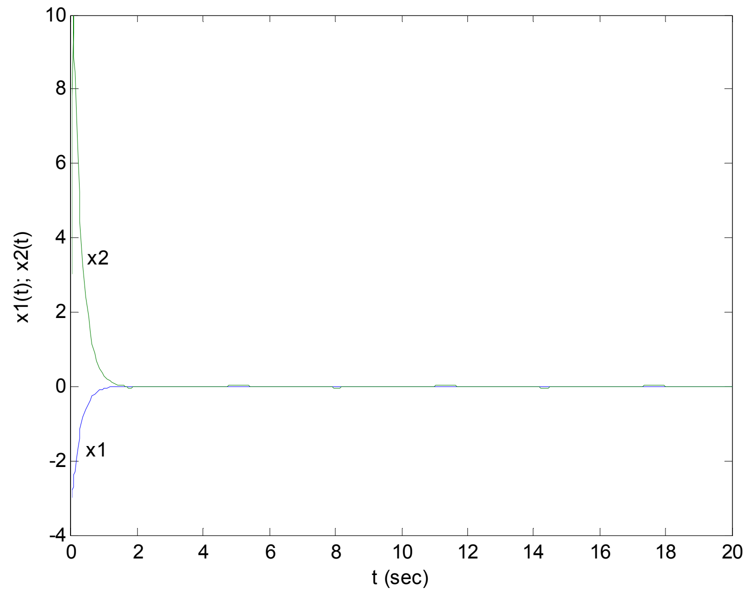

Figure 2: Typical state trajectories of the feedback-controlled system of (9) with (10).

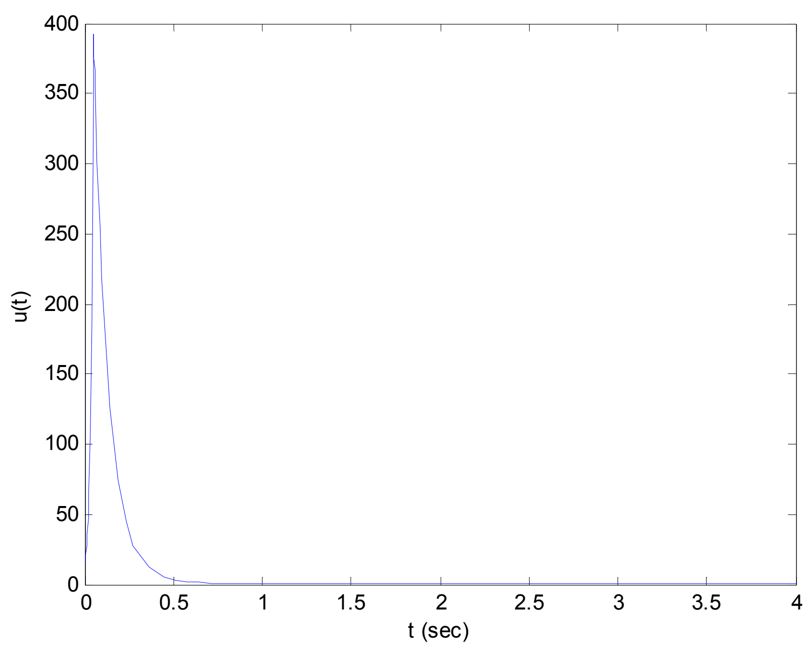

Figure 3: The time response of the control signal.