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Estimation of Parameters of Generalized Geometric Linnik Distribution

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Abstract: *Consider the geometric Linnik distribution GL(α, λ) with characteristic function* $\phi(t)=\dfrac{1}{1+ln(1+\lambda|t|^{\alpha})}, \lambda>0,\ 0<\alpha\le 2.$ *and type II Generalized Geometric Linnik distribution* $GeGL_2(\alpha,\lambda,\nu)$ *with characteristic function* $\phi(t)=\left[\dfrac{1}{1+\ln(1+\lambda|t|^{\alpha})}\right]^{\nu}$. $0<\alpha\le 2$, $\lambda>0$, $\nu>0$. *[9] used empirical characteristic function to estimate the parameters of a stable law. [1]used characteristic function technique to estimate the parameters of geometric stable law (see also, [2]). Here we estimate the parameters of geometric Linnik distribution and Generalized Geometric Linnik distribution using empirical characteristic function.*

Keywords: Geometric Linnik Distribution, Generalized Geometric Linnik Distribution

I. INTRODUCTION

As a generalization of the Linnikdistribution [8] introduced semi α-Laplace distribution. A random variable X on R has semi α-Laplace distribution if its characteristic function $\phi(t)$ is of the form

$$\phi(t) = \frac{1}{1+|t|^{\alpha}\delta(t)} \tag{1.1}$$

where $\delta(t)$ satisfies the functional equation

$$\delta(t) = \delta\left(p^{1/\alpha}t\right), 0<p<1, 0<\alpha\le 2. \tag{1.2}$$

[7]introduced generalized Linnik law with characteristic function

$$\phi(t) = \frac{1}{\left(1+|t|^{\alpha}\right)^{\nu}}, \quad \nu>0, \quad 0<\alpha\le 2. \tag{1.3}$$

This distribution is known as Pakes generalized Linnik distribution. When $\nu=1$, it reduces to α-Laplace distribution where as when $\alpha=2$, it reduces to the generalized Laplacian distribution of [6] .

DEFINITION 1.1

A random variable X on R has the generalized Linnik distribution and write $X \underline{\underline{d}}\, GeL(\alpha,\lambda,p)$ if it has the characteristic function

$$\phi(t)=\frac{1}{(1+\lambda|t|^{\alpha})^{p}}, \ p>0, \lambda>0, 0<\alpha\le 2. \tag{1.4}$$

Geometric Linnik distribution was studied in [3].Type I Generalized Geometric Linnik distribution and type II Generalized Geometric Linnik distribution are introduced by [4].Autoregressive Models of Generalized Geometric Linnik distributions are developed in [5].

II. ESTIMATION OF PARAMETERS OF GEOMETRIC LINNIK DISTRIBUTION

DEFINITION 2.1

A random variable X on R is said to have geometric Linnik distribution and write $X \underset{=}{d} GL(\alpha, \lambda)$ if its characteristic function $\phi(t)$ is

$$\phi(t) \quad = \quad \frac{1}{1+\ln(1+\lambda|t|^{\alpha})}, t \in R, 0<\alpha \leq 2, \lambda>0. \qquad (2.1)$$

[9]used empirical characteristic function to estimate the parameters of a stable law. [1]) used characteristic function technique to estimate the parameters of geometric stable law (see also, [2]). Here we estimate the parameters of geometric Linnik distribution using empirical characteristic function.

Consider the geometric Linnik distribution with characteristic function

$$\phi(t)=\frac{1}{1+ln(1+\lambda|t|^{\alpha})}, \lambda>0,\ 0<\alpha \leq 2.$$

The function $\hat{\phi}_n(t)=\frac{1}{n}\sum_{j=1}^{n} e^{itX_j}$ is called the sample (empirical) characteristic function. We have $E\left[\hat{\phi}_n(t)\right]=\phi(t)$ and by the strong law of large numbers, $\hat{\phi}_n(t) \xrightarrow{a.s} \phi(t)$.

Take

$$\delta(t)=e^{(\frac{1}{\phi(t)}-1)}-1=\lambda|t|^{\alpha}.$$

Then

$$\delta(t_i)=\lambda|t_i|^{\alpha},\ i=1,2.$$

Taking logarithms on both sides, we get

$$\ln\delta(t_1)=\ln\lambda+\alpha\ln|t_1|,$$

$$\ln\delta(t_2)=\ln\lambda+\alpha\ln|t_2|.$$

Hence,

$$\alpha\left[\ln|t_1|-\ln|t_2|\right]=\ln\delta(t_1)-\ln\delta(t_2).$$

That is,

$$\alpha=\frac{\ln\frac{\delta(t_1)}{\delta(t_2)}}{\ln\frac{|t_1|}{|t_2|}}.$$

For $\alpha \neq 1$,

$$\ln\delta(t_1)\ln|t_2|=\ln|t_2|\ln\lambda+\alpha\ln|t_1|\ln|t_2|$$

and

$$\ln\delta(t_2)\ln|t_1| = \ln|t_1|\ln\lambda + \alpha\ln|t_2|\ln|t_1|.$$

Hence,

$$\ln\lambda\{\ln|t_1| - \ln|t_2|\} = \ln\delta(t_2)\ln|t_1| - \ln\delta(t_1)\ln|t_2|$$

Therefore,

$$\lambda = \exp\left\{\frac{\ln\delta(t_2)\ln(t_1) - \ln\delta(t_1)\ln(t_2)}{\ln|t_1| - \ln|t_2|}\right\}.$$

That is, $\hat{\alpha} = \dfrac{\ln\dfrac{\hat{\delta}_n(t_1)}{\hat{\delta}_n(t_2)}}{\ln\dfrac{|t_1|}{|t_2|}}$

and for $\alpha \neq 1$,

$$\hat{\lambda} = \exp\left\{\frac{\ln\hat{\delta}_n(t_2)\ln(t_1) - \ln\hat{\delta}_n(t_1)\ln(t_2)}{\ln|t_1| - \ln|t_2|}\right\}$$

where $\hat{\delta}_n(t) = \exp\left\{\frac{1}{\phi_n(t)} - 1\right\} - 1$ is the sample counterpart of $\delta(t)$.

III. ESTIMATION OF PARAMETERS OF GENERALIZED GEOMETRIC LINNIK DISTRIBUTION

DEFINITION 3.1

A random variable X on R is said to have type I generalized geometric Linnik distribution and write $X \underset{=}{d} GeGL_1(\alpha, \lambda, p)$ if it has the characteristic function

$$\phi(t) = \frac{1}{1 + p\ln(1+\lambda|t|^{\alpha})},\ 0 < \alpha \le 2,\ p > 0,\ \lambda > 0. \qquad (3.1)$$

DEFINITION 3.2

A random variable X on R has type II generalized geometric Linnik distribution and writes $X \underset{=}{d} Ge\,GL_2(\alpha, \lambda, \tau)$, if it has the characteristic function

$$\phi(t) = \left[\frac{1}{1 + ln(1+\lambda|t|^{\alpha})}\right]^{\tau},\ t \in R, 0 < \alpha \le 2,\ \lambda, \tau > 0. \qquad (3.2)$$

Note that when $\tau = 1$, type II generalized geometric Linnik distribution reduces to geometric Linnik distribution.

Following the method of empirical characteristic function used in the case of GL distribution, we can estimate the $GeGL_2$ distribution parameters.

Consider the $GeGL_2(\alpha, \lambda, \nu)$ characteristic function.

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$$\phi(t) = \left[\frac{1}{1+\ln(1+\lambda|t|^{\alpha})}\right]^{\nu}.$$

We have, the empirical characteristic function is

$$\hat{\phi}_n(t) = \frac{1}{n}\sum_{j=1}^{n} e^{itX_j}.$$

$$\ln\phi(t) = -\nu\,\ln\left[1+\ln(1+\lambda|t|^{\alpha})\right].$$

Proceeding as in Section 2, we get

$$\hat{\alpha} = \frac{\ln\frac{\hat{\delta}_n(t_1)}{\hat{\delta}_n(t_2)}}{\ln\frac{|t_1|}{|t_2|}} \text{ and for } \alpha \neq 1,$$

where

$$\hat{\delta}_n(t) = \exp\left(\left[\hat{\phi}_n(t)\right]^{-1/\nu} - 1\right) - 1 \text{ and}$$

$$\hat{\nu} = \frac{-\ln\hat{\phi}_n(t)}{\ln\left[1+\ln(1+\hat{\lambda}|t|^{\hat{\alpha}})\right]}.$$

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