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On Tailed Generalized Geometric Linnik Distributions

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Abstract: *Tailed distributions have found applications in various fields. In this paper, we study tailed generalized geometric Linnik distribution. Relation between stable laws and tailed geometric Linnik distribution is established. Tailed generalized geometric asymmetric Linnik distribution is introduced and studied.*

Keywords: Laplace distribution, Linnik distribution, Stable Laws, Tailed distributions, Geometric Linnik distribution, Generalized Geometric Linnik distribution, Generalized Geometric Asymmetric Linnik distribution.

I. INTRODUCTION

Tailed distributions have found applications in various fields and were studied by many authors (see, [2],[6] and [7]).We encounter tailed distributions in life testing experiments where an item fails instantaneously. In clinical trials, some times a medicine has no response initially with a certain probability and on a later stage there may be response, the length of the response is described by certain probability distribution.

DEFINITION 1.1

Let the random variable X has distribution function F(x) and characteristic function $\phi_X(t)$. A tailed random variable U with tail probability θ associated with X is defined by the characteristic function

$$\phi_U(t) = \theta + (1-\theta)\phi_X(t) \quad (1.1)$$

[1]proved that the function

$$\phi(t) = \frac{1}{1+\lambda|t|^{\alpha}}, 0 < \alpha \le 2, \lambda > 0 \quad (1.2)$$

is the characteristic function of a probability distribution. The distribution corresponding to the characteristic function (1.2) is called Linnik (or α - Laplace) distribution. A random variable X with characteristic function ϕ in (1.2) is denoted by $X \underline{\underline{d}} L(\alpha,\lambda)$. Note that the $L(\alpha,\lambda)$ distributions are symmetric and for $\alpha = 2$, it becomes the classical symmetric Laplace distribution.

[8]introduced a generalization of the Linnik distribution (1.2), namely semi α-Laplace distribution. A random variable X on R has semi α-Laplace distribution if its characteristic function $\phi(t)$ is of the form

$$\phi(t) = \frac{1}{1+|t|^{\alpha}\delta(t)} \quad (1.3)$$

where $\delta(t)$ satisfies the functional equation

$$\delta(t) = \delta\left(p^{1/\alpha}t\right), 0<p<1, 0<\alpha\le 2. \quad (1.4)$$

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DEFINITION 1.2

A random variable X on R is said to have geometric Linnik distribution and write $X \underset{=}{d} GL(\alpha, \lambda)$ if its characteristic function $\phi(t)$ is

$$\phi(t) \quad = \quad \frac{1}{1+\ln(1+\lambda|t|^{\alpha})}, t \in R, 0<\alpha \leq 2, \lambda>0. \qquad (1.5)$$

DEFINITION 1.3

A random variable X on R has the generalized Linnik distribution and write $X \underset{=}{d} GeL(\alpha, \lambda, p)$ if it has the characteristic function

$$\phi(t)=\frac{1}{(1+\lambda|t|^{\alpha})^{p}}, \; p>0, \lambda>0, 0<\alpha \leq 2. \qquad (1.6)$$

DEFINITION 1.4

A random variable X on R is said to have generalized asymmetric Linnik distribution if its characteristic function is

$$\phi(t) \quad = \quad \left(\frac{1}{1+\lambda|t|^{\alpha}-i\mu t}\right)^{\tau}, -\infty<\mu<\infty, \lambda, \tau \geq 0, 0<\alpha \leq 2. \qquad (1.7)$$

We shall denote it by GeAL(α, λ,μ, τ).

Type I and typeII generalized geometric Linnik distributions are are studied in [3] and [4].Generalized geometric asymmetric Linnik distribution is introduced and its properties are discussed in [5].

DEFINITION 1.5

A random variable X on R is said to have type I generalized geometric Linnik distribution and write $X \underset{=}{d} GeGL_1(\alpha, \lambda, p)$ if it has the characteristic function

$$\phi(t)=\frac{1}{1+p\ln(1+\lambda|t|^{\alpha})}, \; 0<\alpha \leq 2, p>0, \lambda>0. \qquad (1.8)$$

DEFINITION 1.6

A random variable X with characteristic function

$$\psi(t)=\frac{1}{1+\tau\ln(1+\lambda|t|^{\alpha}-i\mu t)}, -\infty<\mu<\infty, \lambda, \tau \geq 0, 0<\alpha \leq 2 \qquad (1.9)$$

is called Type I generalized geometric asymmetric Linnik distribution with parameters $\mu, \sigma, \alpha, \tau$ .We represent it as $X \underset{=}{d} GeGAL_1(\alpha, \lambda, \mu, \tau)$.

DEFINITION 1.7

A random variable X on R has type II generalized geometric Linnik distribution and write $X \underset{=}{d} Ge\,GL_2(\alpha, \lambda, \tau)$, if it has the characteristic function

$$\phi(t)=\left[\frac{1}{1+ln(1+\lambda|t|^{\alpha})}\right]^{\tau}, t \in R, 0<\alpha \leq 2, \lambda, \tau>0. \qquad (1.10)$$

Note that when $\tau = 1,$ type II generalized geometric Linnik distribution reduces to geometric Linnik distribution.

In Section 2, we introduce tailed distributions associated with type I generalized geometric Linnik distribution and study its properties. Tailed Type II generalized geometric Linnik distribution is also discussed in this Section. Tailed type I generalized geometric asymmetric Linnik distribution is studied in Section 3.

II. TAILED GENERALIZED GEOMETRIC LINNIK DISTRIBUTIONS

Now we introduce tailed Type I generalized geometric Linnik distribution and obtain a representation of the same.

In (1.1), when $\phi_X(t) = \dfrac{1}{1+\tau \ln\left(1+\lambda|t|^{\alpha}\right)}$, then

$$\phi_U(t) = \frac{1+\tau\theta \ln\left(1+\lambda|t|^{\alpha}\right)}{1+\tau \ln\left(1+\lambda|t|^{\alpha}\right)}. \tag{2.1}$$

The random variable U with characteristic function (2.1) is called tailed Type I generalized geometric Linnik and denoted by $TGeGL_1(\alpha,\lambda,\tau,\theta)$

THEOREM 2.1

Let X and Y be independent random variables such that X has tailed generalized geometric exponential distribution with Laplace transform $\theta+(1-\theta)\dfrac{1}{1+\tau\ln(1+\delta)}$ and Y is stable with characteristic function $e^{-\lambda|t|^{\alpha}}$, $0<\alpha\le 2, \lambda, \delta, \tau > 0,\ 0<\theta<1$. Then $Z = X^{1/\alpha}Y$ has $TGeGL_1(\alpha,\lambda,\tau,\theta)$ distribution.

PROOF

$$\phi_Z(t) = E\left[e^{itX^{1/\alpha}Y}\right]$$

$$= \int_0^{\infty} \phi_Y\left(tx^{1/\alpha}\right) dF(x)$$

$$= \int_0^{\infty} e^{-\lambda|t|^{\alpha}x} dF(x)$$

$$= \theta + (1-\theta)\frac{1}{1+\tau \ln\left(1+\lambda|t|^{\alpha}\right)}$$

$$= \frac{1+\tau\theta \ln\left(1+\lambda|t|^{\alpha}\right)}{1+\tau \ln\left(1+\lambda|t|^{\alpha}\right)}.$$

This completes the proof.

Now we consider the type II generalized geometric Linnik distribution and study the tailed distribution generated by it.

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DEFINITION 2.1

A random variable X is said to have tailed type II generalized geometric Linnik distribution and write $X \underset{=}{d} TGeGL_2(\alpha,\lambda,\tau,\theta)$ distribution if it has the characteristic function

$$\phi_X(t) = \frac{\theta\left[1+\ln\left(1+\lambda|t|^{\alpha}\right)\right]^{\tau}+(1-\theta)}{\left[1+\ln\left(1+\lambda|t|^{\alpha}\right)\right]^{\tau}}, 0<\alpha\le 2, 0<\theta<1, \lambda,\tau>0.$$

The tailed type II generalized geometric Linnik distribution being the tailed form of type II generalized geometric Linnik is infinitely divisible.

As in the case of $TGeGL_1$ distribution, we now obtain a representation of $TGeGL_2$ random variables in terms of tailed geometric gamma and stable random variables.

DEFINITION 2.2

A random variable X is said to have tailed geometric gamma distribution if it has Laplace transform

$$\phi_1(\delta) = \theta + (1-\theta)\frac{1}{\left[1+\ln(1+\delta)\right]^{\tau}}, \quad \delta,\tau>0, 0<\theta<1.$$

THEOREM 2.2

Let X and Y be independent random variables such that X has Laplace transform $\theta+(1-\theta)\frac{1}{[1+\ln(1+\delta)]^{\tau}}$ and Y is stable with characteristic function $e^{-\lambda|t|^{\alpha}}, 0<\alpha\le 2$. Then $U = X^{1/\alpha}Y$ has distribution $TGeGL_2(\alpha,\lambda,\tau,\theta)$.

Proof follows analogous to the proof of Theorem 2.1.

III. TAILED GENERALIZED GEOMETRIC ASYMMETRIC LINNIK DISTRIBUTION

Here we introduce and study tailed distributions generated by generalized geometric asymmetric Linnik distributions.

DEFINITION 3.1

A random variable U is said to have tailed type I generalized geometric asymmetric Linnik distribution and write $U \underset{=}{d} TGeGAL_1(\alpha,\lambda,\mu,\tau,\theta)$ if it has characteristic function

$$\phi_U(t) = \frac{1+\tau\theta\ln\left(1+\lambda|t|^{\alpha}-i\mu t\right)}{1+\tau\ln\left(1+\lambda|t|^{\alpha}-i\mu t\right)}.$$

The type II generalized geometric asymmetric Linnik distribution can be defined using (1.1) with $\phi_X(t)$ replaced by

$$\frac{1}{\left[1+\ln\left(1+\lambda|t|^{\alpha}-i\mu t\right)\right]^{\tau}}.$$

That is, a random variable U having tailed type II generalized geometric asymmetric Linnik distribution denoted by $TGeGAL_2(\alpha,\lambda,\mu,\tau,\theta)$ has characteristic function

$$\theta+(1-\theta)\frac{1}{\left[1+\ln\left(1+\lambda|t|^{\alpha}-i\mu t\right)\right]^{\tau}}.$$

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