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Necessary and Sufficient Conditions for Ensuring Stability and Avoiding Sinusoidal Oscillation of Uncertain Interval Systems

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Abstract*: In this paper, two types of second-order uncertain interval systems are proposed and discussed. Based on control theory, time-domain approach, and bilinear transformation method, the necessary and sufficient conditions are derived for above two interval systems to ensure that stability can be achieved and that the output signal will not produce sinusoidal oscillations. Finally, several numerical simulations are given to illustrate the feasibility and effectiveness of the obtained results.*

Keywords: Sinusoidal oscillation, Uncertain systems, Continuous-data interval systems, Discrete-data interval systems

I. INTRODUCTION

Most systems contain uncertainties to some extent, such as unknown system parameters or hard-to-estimate noise. Interval systems can often correctly describe a reasonable model containing uncertain factors. There is no doubt that systems containing uncertain factors are more difficult to analyze and design than systems without uncertainty factors.

In the past few years, various interval systems have not only been proposed, but also extensively studied; see, for example, [1]-[9] and the references therein. In [3], the interval observer control methodology has been applied to reduce the influence of unknown external disturbances. By introducing sliding mode term, the observation error of the system has been released and the robustness has been ameliorated. In [4], a multiple cyber-attack model has been constructed for the interval type-2 fuzzy systems with multiple cyber-attacks and external disturbances. Moreover, by establishing a proper Lyapunov-Krasovskii functional, sufficient conditions have been deduced to ensure the exponentially mean-square stable of the interval type-2 fuzzy systems with the H_∞ performance.

This paper will focus on two types of second-order uncertain interval systems and discuss the necessary and sufficient conditions to ensure that stability can be achieved and that the output signal will not produce sinusoidal oscillations. Finally, some numerical simulation examples will be provided to illustrate the practicality and correctness of the main results. Throughout this paper, $\Re$ denotes the real numbers, $|a|$ represents the modulus of a complex number *a*, and $[a, b]$ represents the set of $\{x | a \le x \le b, x \in \Re\}$.

II. UNCERTAIN CONTINUOUS-DATA INTERVAL SYSTEMS

Consider the following uncertain continuous-data interval systems:

$$\Delta a_2 \cdot \frac{d^2 y(t)}{dt^2} + \Delta a_1 \cdot \frac{dy(t)}{dt} + \Delta a_0 \cdot y(t) = 0, \qquad (1)$$

where $y(t)$ is the output signal and Δa_i are uncertain parameters of the uncertain systems, with $\Delta a_2 \in [\underline{a_2}, \overline{a_2}]$, $\Delta a_1 \in [\underline{a_1}, \overline{a_1}]$, $\Delta a_0 \in [\underline{a_0}, \overline{a_0}]$, and $0 \notin [\underline{a_2}, \overline{a_2}]$. From a practical point of view, the uncertain interval

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system just reflects the reasonable model of most real dynamic systems; on the other hand, a stable system without sine wave oscillation at the output end of the system is often the dream target of control engineers.
The following are equivalent criteria to ensure that system (1) is a stable system and the output does not undergo sinusoidal oscillations.

Theorem 1: Uncertain continuous-data interval systems(1)are stable systems the output will not oscillate with sinusoidal waves if and only if the following conditions are met simultaneously:

(c1) $\Delta a_2, \Delta a_1$ and Δa_0 have the same sign, $\forall\ \Delta a_i \in \left[\underline{a_i}, \overline{a_i}\right]$ and $i \in \{0, 1, 2\}$;

(c2) $\min\limits_{\Delta a_i \in \left[\underline{a_i}, \overline{a_i}\right], i \in \{0,1,2\}} \left[\Delta a_1^2 - 4\Delta a_2 \Delta a_0\right] \geq 0$.

Proof.

(i) Obviously, the characteristic equation of system (1) is $\Delta a_2 s^2 + \Delta a_1 s + \Delta a_0 = 0$ and the necessary and sufficient conditions to ensure that it is a stable system are that $\Delta a_2, \Delta a_1$ and Δa_0 have the same sign, in view of Routh-Hurwite criterion [10] with Routh's tabulation:

s^2	Δa_2	Δa_0
s^1	Δa_1	
s^0	Δa_0	

(ii) The output of the system (1) will not produce sinusoidal oscillations.

$\Leftrightarrow$ Both roots of the quadratic equation $\Delta a_2 s^2 + \Delta a_1 s + \Delta a_0 = 0$ are real.

$\Leftrightarrow$ $\Delta a_1^2 - 4\Delta a_2 \Delta a_0 \geq 0$, $\forall\ \Delta a_i \in \left[\underline{a_i}, \overline{a_i}\right]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ $\min\limits_{\Delta a_i \in \left[\underline{a_i}, \overline{a_i}\right], i \in \{0,1,2\}} \left[\Delta a_1^2 - 4\Delta a_2 \Delta a_0\right] \geq 0$.

From the above (i) and (ii), this proof is completed.
The following provides an example to illustrate the application of Theorem 1.

Example 1: Consider the following uncertain continuous-data interval systems:

$$\frac{Y(s)}{U(s)} = \frac{\Delta b_4}{\Delta b_1 \cdot s^2 + \Delta b_2 \cdot s + \Delta b_3}, \quad (2a)$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of the output and input respectively, and Δb_i are uncertain parameters of such interval systems, with

$$\Delta b_1 \in [1, 3],\ \Delta b_2 \in [-3, -1],\ \Delta b_3 \in [4, 6],\ \Delta b_4 \in [2, 4]. \quad (2b)$$

It is known from Theorem 1 that system (2) is an unstable system and the output will oscillate with sinusoidal waves.Below we will design a PD controller $U(s) = (K_p + K_d s)Y(s) + R(s)$ to make the entire closed-loop control system become a stable system and the output will not oscillate with sinusoidal waves.It is worth mentioning that K_p and K_d are parameters to be designed, and the block diagram of the entire closed-loop system is shown in Figure 1.So far, the characteristic equation of the entire closed-loop control system is $\Delta b_1 \cdot s^2 + (\Delta b_2 + \Delta b_4 K_d) \cdot s + (\Delta b_3 + \Delta b_4 K_p) = 0$. It can be deduced from Theorem 1 that if any one of the following conditions (c3) and (c4) is met, the entire closed-loop system will be a stable system and the output will not oscillate with sinusoidal waves.

(c3) $K_d > 3\sqrt{2} + \frac{3}{2}$ and $0 < K_p \leq \frac{(-3 + 2K_d)^2}{48} - \frac{3}{2}$

(c4) $-1 < K_p < 0$ and $K_d \geq 36 + 12K_p + \frac{3}{2}$

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It is obvious that $(K_p, K_d) = (0.1, 6)$ satisfies condition (c3). The output signal diagrams without a controller and with PD controller $U(s) = (0.1 + 6s)Y(s) + R(s)$ are shown in Figure 2 and Figure 3, respectively. Figure 2 does show instability and sine wave oscillation; however, Figure 3 shows a stable system without sine wave oscillation.

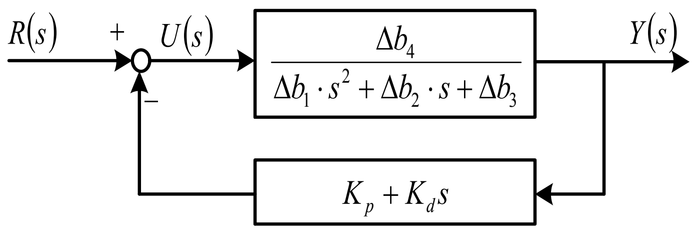

Figure1: Block diagram of uncertain interval control system.

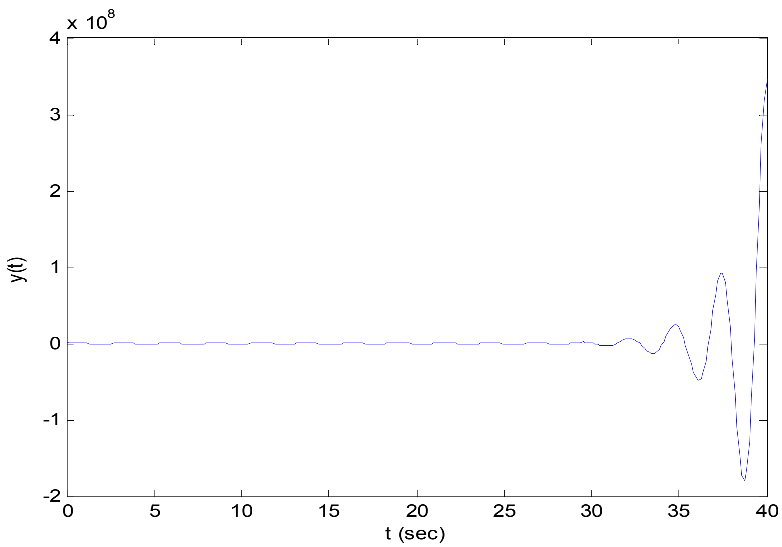

Figure 2: Typical output trajectory of the uncontrolled system of Example 1.

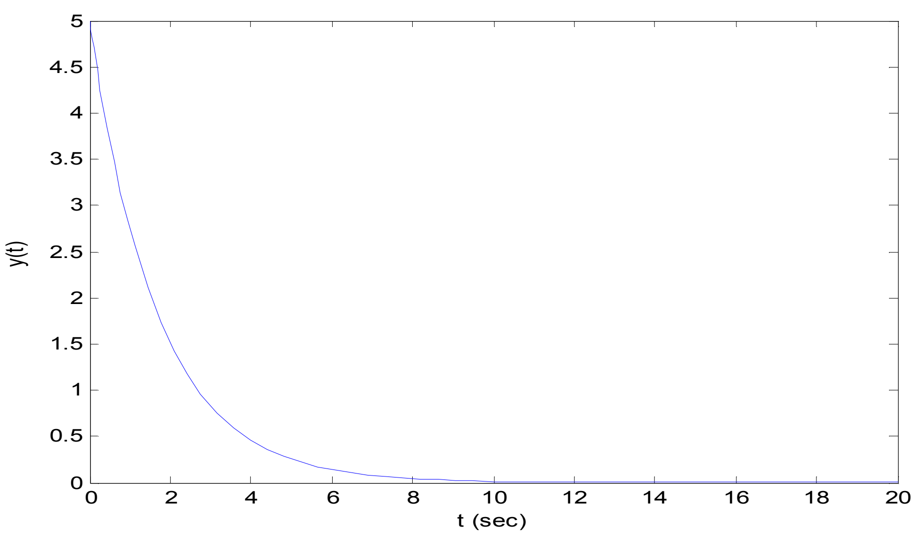

Figure 3: Typical output trajectory of the feedback-controlled system of Example 1.

III. UNCERTAIN DISCRETE-DATA INTERVAL SYSTEMS

Consider the following uncertain discrete-data interval systems:

$$\Delta a_2 \cdot y(n+2) + \Delta a_1 \cdot y(n+1) + \Delta a_0 \cdot y(n) = 0, \quad \forall\, n \in N, \qquad (3)$$

where $y(n)$ is the output signal and Δa_i are unknown parameters of the systems, with $\Delta a_2 \in [\underline{a_2}, \overline{a_2}]$, $\Delta a_1 \in [\underline{a_1}, \overline{a_1}]$, $\Delta a_0 \in [\underline{a_0}, \overline{a_0}]$, and $0 \notin [\underline{a_2}, \overline{a_2}]$. As we know, in actual dynamic systems, most do not want instability to occur, and it is expected that the output signal will not oscillate as a sinusoidal wave.Similarly, this section will explore the criteria to ensure that the above uncertain interval systems become stable systems and no sinusoidal oscillation occurs at the output.

Another result of this paper is presented as follows.

Theorem 2: The discrete uncertain discrete-data interval systems(3) are stable systems and the output signal will never oscillate as a sinusoidal wave if and only if the following conditions are met at the same time:

(c5) $\Delta a_2 + \Delta a_1 + \Delta a_0$, $\Delta a_2 - \Delta a_0$, and $\Delta a_2 - \Delta a_1 + \Delta a_0$ have the same sign, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$;

(c6) $\min_{\Delta a_i \in [\underline{a_i}, \overline{a_i}],\, i \in \{0,1,2\}} \left[\Delta a_1^2 - 4\Delta a_2 \Delta a_0\right] \ge 0$.

Proof. (i) It is easy to see that the characteristic equation of system (3) is $f(z) := \Delta a_2 z^2 + \Delta a_1 z + \Delta a_0 = 0$. By combining the equation $f(z) = 0$ with the bilinear transformation $z = \frac{s+1}{s-1}$ [10], we can get

$$g(s) := (\Delta a_2 + \Delta a_1 + \Delta a_0)s^2 + 2(\Delta a_2 - \Delta a_0)s + (\Delta a_2 - \Delta a_1 + \Delta a_0) = 0.$$

Thus one has

$\Delta a_2 + \Delta a_1 + \Delta a_0$, $\Delta a_2 - \Delta a_0$, and $\Delta a_2 - \Delta a_1 + \Delta a_0$ have the same sign, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ $\Delta a_2 + \Delta a_1 + \Delta a_0$, $2(\Delta a_2 - \Delta a_0)$, and $\Delta a_2 - \Delta a_1 + \Delta a_0$ have the same sign, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ The real parts of both roots of function $g(s)$ are less than zero, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ If z is the root of the function $f(z)$, then $|z| < 1$ must be true, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ The uncertain discrete-data interval systems(3) are stable systems, in view of bilinear transformation method [10] and the first part of the proof of Theorem 1.

(ii) The output of systems(3) will not undergo sinusoidal oscillation.

$\Leftrightarrow$ Both roots of the quadratic equation $\Delta a_2 s^2 + \Delta a_1 s + \Delta a_0 = 0$ are real.

$\Leftrightarrow$ $\Delta a_1^2 - 4\Delta a_2 \Delta a_0 \ge 0$, $\forall\, \Delta a_i \in [\underline{a_i}, \overline{a_i}]$ and $i \in \{0, 1, 2\}$.

$\Leftrightarrow$ $\min_{\Delta a_i \in [\underline{a_i}, \overline{a_i}],\, i \in \{0,1,2\}} \left[\Delta a_1^2 - 4\Delta a_2 \Delta a_0\right] \ge 0$.

This proof is completed based on the above (i) and (ii).

A numerical example is given below to illustrate Theorem 2.

Example 2: Consider the following uncertain discrete-data interval systems:

$$\Delta a_2 \cdot y(n+2) + \Delta a_1 \cdot y(n+1) + \Delta a_0 \cdot y(n) = 0, \quad \forall\, n \in N, \qquad (4a)$$

where $y(n)$ is the output signal and Δa_i are uncertain parameters of above interval systems, with

$$\Delta a_2 \in [14, 16],\ \Delta a_1 \in [-10, -8],\ \Delta a_0 \in [-1, 1]. \qquad (4b)$$

Clearly, we have $\Delta a_2 + \Delta a_1 + \Delta a_0 \in [3, 9]$, $\Delta a_2 - \Delta a_0 \in [13, 17]$, $\Delta a_2 - \Delta a_1 + \Delta a_0 \in [21, 27]$, and $\Delta a_1^2 - 4\Delta a_2 \Delta a_0 \ge \min\left[\Delta a_1^2 - 4\Delta a_2 \Delta a_0\right] = 64 - 64 \ge 0$. This means that both (c5) and (c6) are true.

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Consequently, by Theorem 2, we conclude thatuncertain discrete-data interval systems of (4) are stable systems and the output signal will never oscillate as a sinusoidal wave. The following is a further analysis of two special cases of System (4). In case of $14y(n+2)-10y(n+1)-y(n)=0, \forall\, n \in N$, with $[y(1) \quad y(2)] = [3 \quad 2]$, the output signal is shown in Figure 4; and in the case of $16y(n+2)-8y(n+1)+y(n)=0, \forall\, n \in N$, with $[y(1) \quad y(2)] = [4 \quad -3]$, the output signal is shown in Figure 5.It can be seen from the above two figures that the systems in these two cases are not only stable systems, but also the output signals do not undergo sinusoidal oscillation.

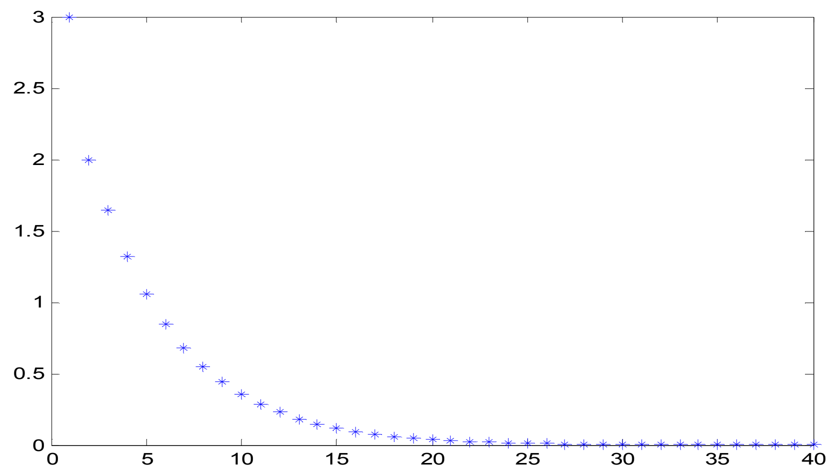

Figure 4: Output trajectory of $14y(n+2)-10y(n+1)-y(n)=0, \forall\, n \in N$, with $[y(1) \quad y(2)] = [3 \quad 2]$.

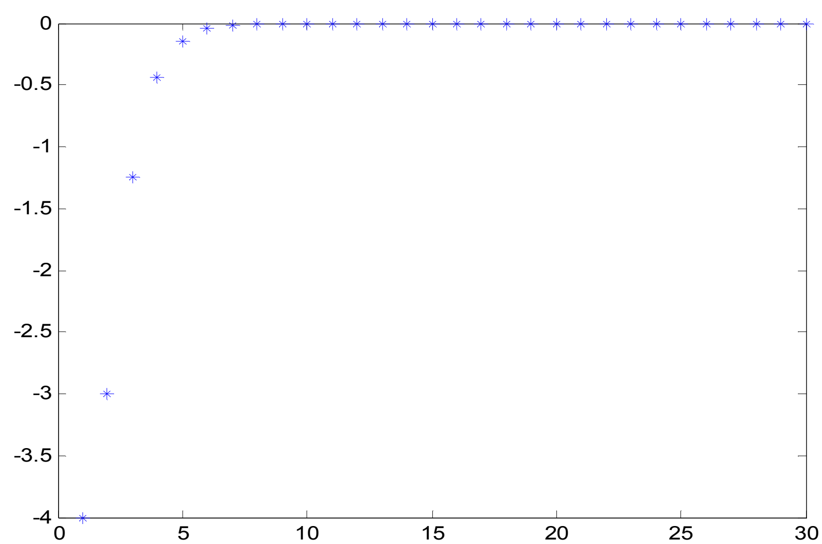

Figure 5: $16y(n+2)-8y(n+1)+y(n)=0, \forall\, n \in N$, with $[y(1) \quad y(2)] = [4 \quad -3]$.

IV. CONCLUSION

In this paper, two types of second-order uncertain interval systems have been proposed and discussed. Based on control theory, time-domain approach, and bilinear transformation method, the necessary and sufficient conditions have been derived for above two interval systems to ensure that stability can be achieved and that the output signal will not produce sinusoidal oscillations. Finally, several numerical simulations have been offered to illustrate the correctness and feasibility of the main results.

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