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Study on Finding The Roots of Non-Linear Equations using a Parallel Hybrid Algorithm of the Bisection and Newton-Raphson Methods

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Abstract: *With inspiration from the Bisection method and the Newton-Raphson algorithm, this work presents a novel parallel Hybrid algorithm. Finding the true roots of a single non-linear equation using the suggested Hybrid algorithms takes less time and requires less iterations of the procedure. We have used these strategies in a parallel setting. In this paper, we discuss each method and compare them to one another.*

Keywords: Hybrid Algorithm, bisection Method, Parallel Hybrid Algorithm, Parallel Numerical Algorithm, Newton-Raphson Method

I. INTRODUCTION

Many applications in mathematics, computer science, Physics and engineering are interested in parallelism-related subjects. The majority of algorithms are sequential, meaning that each step in the sequence only involves one operation. These algorithms work well on modern computers, which essentially carry out operations sequentially. Even if the speed at which sequential computers function has increased exponentially over the years, the expense of the advancement is rising. Researchers have thus looked for more affordable advancements [4].

The focus in algorithm design has switched from sequential algorithms to parallel algorithms, that is, algorithms in which many processes are executed concurrently [5], as more computers have implemented some sort of parallelism. Without outlining the methodology, Maeder supplied a parallel Newton-Raphson method diagram. They obtained the findings by simulating parallel processes running while a serial application was being executed on a typical SISD VAX 11-780 machine.

Operating system utilities that were independently tested for consistency and dependability were used to measure the execution times of parallel processes. A two processor approach was employed for single-root methods, including parallel Newton-Raphson and secant method implementations. [1]

By using three sequences an, bn, and cn each of which is generated by a different processor Ioana was able to master the serial algorithm of the Bisection technique for parallel execution with three processors [2]. She took into account that the actual function f(x) has only one zero in the range [a,b].

In this work, we provide an unique parallel hybrid strategy for locating actual function roots, as well as a novel parallelization technique for the bisection method. The sequence is as follows: Section 2 introduces root-finding, Bisection, Newton-Raphson, and Hybrid methods in sequential algorithms; Section 3 describes a novel method for converting a sequential numerical algorithm to a parallel algorithm using Newton-Raphson, Bisection, and Hybrid methods; and Section 4 concludes with results and discussions.

Root-finding Method

One of the most difficult problems in computing is said to be the root-finding issue. Engineering, physics, chemistry, and biology are only a few of the many real-world applications that it grows into. In actuality, anytime an implicit unknown included in technical or scientific formulations is discovered, a root-finding difficulty arises. A root-finding algorithm is a numerical technique for finding a value for a function f such that $f(x) = 0$ is the outcome. Iteration is a

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technique used to generate a series of numbers that should eventually converge on a limit that is a root in numerical root-finding techniques. This limit is called the "fixed point," and because it is a limit, it is called the "fixed point." These series' starting values are based on educated estimations. The procedure uses both the function and the current values in order to determine the new values.

Bisection Method:

The Bisection (Binary Search) method is based on the Intermediate Value Theorem (IVT). In mathematics, the bisection method is a strategy for identifying roots that repeatedly bisects an interval before selecting a subinterval within which a root must lie. Although it is a basic and trustworthy process, it is also quite slow. As a consequence, it is often employed to offer an approximation that serves as a foundation for algorithms that converge more rapidly. This approach is sometimes referred to as the binary search method, the dichotomy method, and the interval halving method [6][7][8]. f(x) is continuous on [a,b] if f(a) and f(b) both have the opposite sign.

Newton – Raphson Method:

Newton-Raphson technique is a widely used numerical approach in numerical analysis for finding increasingly improved approximations to the zeros of a real-valued function f (x) = 0 .[9]

$X_{n+1} = x_n - [f (x_n) / df(x_n)]$ (1)

This approach differs from others in that it needs the evaluation of both the function f(x) and its derivative f'(x) at random x coordinates.

Hybrid Algorithm [3]

This algorithm offers a unique approach for obtaining the roots of nonlinear equations with the sign f(x)=0 by combining the Newton-Raphson algorithm with the Bisection algorithm. A first approximation may be obtained by utilizing the Bisection technique twice, and a correct approximation can be obtained by using the Newton-Raphson method.

Given f ,df , a , b , and δ=10-6 (tolerance)

Parallel Methods

In this part, we provide a novel method for transforming sequential numerical algorithms into parallel ones.

Parallel Newton-Raphson Algorithm

Meaider [1] introduced Parallel Newton-Raphson Method with a graphic, but he did not explain what it was. Here is how we describe the parallel algorithm.

Parallel Bisection Method:

By distributing the k^{th} intervals of roots over the kth cores, we proposed a novel method for parallelizing the Bisection algorithm. Parallelism was also used to find the images of these interval sends.

Given f , a[k] , b[k] , and δ (tolerance)

Step 1 : do in parallel for four roots

Step 2 : i=1

Step 3 : c = (a[k] + b[k]) / 2

Step 4 : : compute f(c) and f(a[k]) in parallel

Step 5 : If f (c) = 0 or f (c) < δ , then step 9

Step 6 : If f (a[k]) * f (c) < 0 , then b[k] = c

Step 7 : Else a[k] = c

Step 8 : i = i + 1 , go back to step 3

Step 9 : stop iteration .

Parallel Hybrid algorithm

We parallelize the hybrid algorithm, which uses two approximations, by distributing the first approximation in kth intervals for kth cores with finding images of intervals ends in parallel. We then incorporate the results of the first parallel algorithm into the second parallel approximation by performing the same task in parallel for f(x) and df(x), respectively.

Given f ,df , a[k] , b[k] and δ (tolerance)
Step 1: do in parallel for four roots
Step 2: for i=1 to 2
Step 3: xi = (a[k] + b[k]) / 2
Step 4: compute f(xi) and f(a[k]) in parallel
Step 5: If f (xi) = 0 or f (xi) < δ , then step 15
Step 6: If f (a) * f (xi) < 0, then b[k] = xi
Step 7: Else a[k] = xi
Step 8: end for
Step 9: for i = 1 to n
Step 10: do in parallel compute f (xi) and df(xi)
Step 11: x = xi - f (xi) / df(xi)
Step 12: If f (xi) < δ , then go to step 15
Step 13: xi = x
Step 14: end for
Step 15: stop iteration.

II. RESULTS AND DISCUSSIONS

Using Matlab, we put the parallel numerical techniques into use on a Dell i7 core Intel (4 cores) machine.

The quartic function case study is real function as $f(x)= a x^4 + b x^3 + c x^2 + d x + e$,we take as for example $f(x)= x^4 +4x^3 – 16x^2 -4^x + 8$,with tolerance $\delta=10^{-6}$, where i= number of iterative, a & b are initials values and c= (a + b) /2.

While parallel Newton-Raphson and parallel Hybrid provide convergent solutions, parallel Hybrid surpasses parallel bisection in terms of iterations and overall computing time.

The serial Bisection method needs to evaluate, add, and multiply the log2 [(b-a)/] function in order to encircle the zero in an interval of length [2]. The number of operations while utilizing a parallel bisection method is 1/m log 2 [(b-a)/], where (m) is the system's processor count. The serial Newton-Raphson technique requires log2, where is the quantity of iterations required to get at f. (x) Therefore, the number of operations required by the hybrid approach is log2 [(b-a) /] + log2, and the number of operations required by the hybrid method in parallel is 1/m log2[(b-a) /] + log2, where m is the number of cores employed in the parallel system.

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