

Heat and Mass Transfer Analysis in An Unsteady Natural Convective Magnet Hydrodynamic Flow of a Nano Fluid under the Presence of Thermal Diffusion and Absorption Radiation

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Abstract: *An analysis of unsteady natural convective MHD flow of nanofluid under the presence of thermal diffusion and absorption radiation with the effects of chemical reaction on heat and mass transfer without a porous medium is discussed in the present study. The governing equations were implied and were solved analytically using perturbation technique. The velocity, temperature and concentration fields are obtained. Graphical results were presented for velocity, temperature and concentration profile for various values of parameters.*

Keywords: Nanofluids, Natural convection, Thermal diffusion.

I. INTRODUCTION

The study of unsteady natural convectional flow of a Nano fluid past over a vertical permeable semi-infinite plate moving with the constant heat source under various effects has attracted the attention of a number of researchers, because of its possible applications in Automobile, solar energy, Mechanical and electronics cooling. In view of these applications, a serious of investigations have been done by various researchers to study the problem of the free natural convective flow of a Nano fluid over a past vertical permeable semi-infinite plate moving with constant heat source. In this dissertation we have made an attempt to study the effects of Diffusion thermo, radiation absorption and chemical reaction of Nano fluids on the Magnetohydrodynamics unsteady natural convective heat and mass transfer flow bounded by semi-infinite plate without porous medium. A constant velocity u_0 is applied when the plate is moved. A magnetic field is applied uniformly along the y- direction. The temperature and concentration profiles are assumed to be varying with respect to time at the plate. Analytical expressions were obtained. Using these expressions, calculating their behavior and the different flow characteristics for various non-dimensional parameters are discussed.

II. MATHEMATICAL FORMULATION

We have considered two dimensional unsteady natural convectional flow of a nanofluid past over a vertical permeable semi-infinite movable plate with heat source as constant. The X - axis is acting vertically upwards which is taken along the plate and the y- direction is perpendicular to the plate. A uniform magnetic field of strength B_0 is applied externally along the y- direction. The temperature T'_∞ remains constant for the plate and the fluid. The concentration C'_∞ at the stationary condition is taken constant in both the fluid and the plate. The fluid we consider here is water based Copper Nano particles and Titanium oxide. Then it is assumed for both the fluid phase nanoparticles are in thermal equilibrium state as they have uniform size and shape.

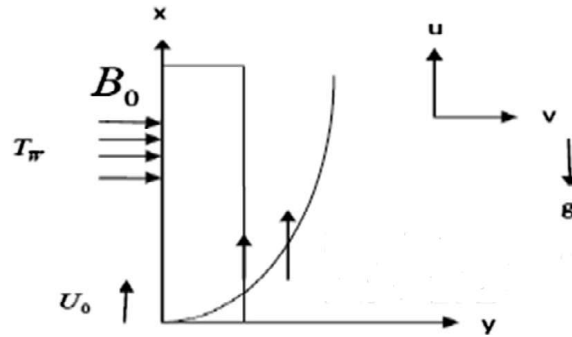


Figure 1: Schematic diagram of the physical properties

From the above assumptions the equation that govern the two dimensional unsteady natural free convective MHD flow of heat and mass transfer analysis for the Nano fluid occupying the plate are given below.

The Continuity equation

$$\frac{\partial v'}{\partial y'} = 0. \quad (3.2.1)$$

The Momentum equations

$$\rho_{nf} \left(\frac{\partial v'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \mu_{nf} \frac{\partial^2 u'}{\partial y'^2} + (\rho\beta)_{nf} g(T' - T'_{\infty}) - \sigma B_0^2 u' \quad (3.2.2)$$

The Energy equation

$$\left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = \alpha_{nf} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q'}{(\rho c p)_{nf}} (T' - T'_{\infty}) + Q' (C' - C'_{\infty}) + \frac{D_m K_T}{c_s (\rho c p)_{nf}} \cdot \frac{\partial^2 C'}{\partial y'^2} \quad (3.2.3)$$

The Species equation

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D_B \frac{\partial^2 C'}{\partial y'^2} - K_l (C' - C'_{\infty}) \quad (3.2.4)$$

The boundary conditions are given by,

$$t' < 0, u'(y', t') = 0, T' = T'_{\infty}, C' = C'_{\infty}$$

$$t' \geq 0, u'(y', t') = U_0, T' = T'_w + (T'_w - T'_{\infty}) \varepsilon e^{iw't'}, C' = C'_w + (C'_w - C'_{\infty}) \varepsilon e^{iw't'} \text{ at } y' = 0$$

$$u'(y', t') = 0, T' = T'_{\infty}, C' = C'_{\infty} \text{ as } y' \rightarrow \infty$$

All physical and parameter are defined in the nomenclature section

Then ,

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$$

$$k_{nf} = k_f \left(\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \right),$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad (3.2.5)$$

$$(\rho c p)_{nf} = (1 - \phi)(\rho c p)_f + \phi(\rho c p)_s,$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c p)_{nf}} \quad (3.2.6)$$

$$v' = -V_0 \quad (3.2.7)$$

Where $-V_0$ constant indicates the normal velocity at the plate that is the positive suction ($V_0 > 0$) and the negative suction represents the blowing injection

$$(V_0 < 0).$$

By applying 3.2.7 in (3.2.2) – (3.2.4), we get

$$\rho_{nf} \left(\frac{\partial v'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} \right) = \mu_{nf} \frac{\partial^2 u'}{\partial y'^2} + (\rho\beta)_{nf} g(T' - T'_{\infty}) - \sigma B_0^2 u' \quad (3.2.8)$$

$$\left(\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} \right) = \alpha_{nf} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q'}{(\rho c p)_{nf}} (T' - T'_{\infty}) + Q' (C' - C'_{\infty}) + \frac{D_m K_T}{c_s (\rho c p)_{nf}} \cdot \frac{\partial^2 C'}{\partial y'^2} \quad (3.2.9)$$

$$\frac{\partial c'}{\partial t'} - v_0 \frac{\partial c'}{\partial y'} = D_B \frac{\partial^2 c'}{\partial y'^2} - K_l(C' - C'_\infty) \quad (3.2.10)$$

Substitute (3.2.6) in (3.2.9)

$$\left(\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'}\right) = \frac{k_{nf}}{(\rho c p)_{nf}} \cdot \frac{\partial^2 T'}{\partial y'^2} - \frac{Q'}{(\rho c p)_{nf}}(T' - T'_\infty) + Q'(C' - C'_\infty) + \frac{D_m K_T}{c_s(\rho c p)_{nf}} \cdot \frac{\partial^2 C'}{\partial y'^2} \quad (3.2.11)$$

$$\rho_{nf} \left(\frac{\partial v'}{\partial t'} - v_0 \frac{\partial v'}{\partial y'}\right) = \mu_{nf} \frac{\partial^2 u'}{\partial y'^2} + (\rho\beta)_{nf} g(T' - T'_\infty) - \sigma B_0^2 u' \quad (3.2.12)$$

$$\frac{\partial c'}{\partial t'} - v_0 \frac{\partial c'}{\partial y'} = D_B \frac{\partial^2 c'}{\partial y'^2} - K_l(C' - C'_\infty) \quad (3.2.13)$$

We define the following dimensionless variables

$$u = \frac{u'}{U_0}, y = \frac{U_0 y'}{v_f}, t = \frac{U_0^2 t'}{v_f}, \omega = \frac{v_f \omega'}{U_0^2}, \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}$$

$$M = \frac{\sigma B_0^2 v_f}{\rho_f U_0^2}, Du = \frac{D_m K_T (C'_w - C'_\infty)}{K_f c_s (T'_w - T'_\infty)}, Q_L = \frac{Q'_l (C'_w - C'_\infty)}{U_0^2 (T'_w - T'_\infty)}, Kr = \frac{k_f v_f}{U_0^2},$$

$$Sc = \frac{v_f}{D_B}, Q = \frac{Q' v_f^2}{K_f U_0^2}, Pr = \frac{v_f}{\alpha_f}, K = \frac{k' \rho_f U_0^2}{v_f^2},$$

$$Gr = \frac{(\rho\beta)_{nf} g v_f (T'_w - T'_\infty)}{\rho_f U_0^3}, \Psi = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)}, S = \frac{v_0}{U_0}.$$

The governing equation (3.2.11)-(3.2.13) together with the dimensionless variables becomes:

$$A \left(\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial y}\right) = D \frac{\partial^2 u}{\partial y^2} + BGr\theta - Mu = 0. \quad (3.2.14)$$

$$C \left(\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial y} - Q_L \Psi\right) = \frac{1}{Pr} \left(E \frac{\partial^2 u}{\partial y^2} - Q\theta\right) + \frac{Du}{Pr} \cdot \frac{\partial^2 \Psi}{\partial y^2} \quad (3.2.15)$$

$$\frac{\partial \Psi}{\partial t} - S \frac{\partial \Psi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \Psi}{\partial y^2} - Kr\Psi \quad (3.2.16)$$

The boundary conditions are given by

$$t < 0: u=0, \theta = 0, \Psi = 0$$

$$t \geq 0: u=1, \theta = 1 + \varepsilon e^{i\omega t}, \Psi = 1 + \varepsilon e^{i\omega t}, \text{ at } y=0$$

$$u=0, \theta = 0, \Psi=0 \text{ as } y \rightarrow \infty$$

III. SOLUTION OF THE PROBLEM

Equations 3.2.14 – 3.2.16 are joined non-linear partial differential equations which are in the closed-form whose solutions are difficult to be obtained. To obtain the solution of the equations, we are converting the non-linear partial differential equations into ordinary

differential equations. The expressions for velocity, temperature and concentration are considered as follows, for the reason that the unsteady flow is placed on the mean steady flow in the neighbourhood of the plate .

$$u(y, t) = u_0 + \varepsilon u_1 e^{i\omega t} \quad (3.2.17)$$

$$\theta(y, t) = \theta_0 + \varepsilon \theta_1 e^{i\omega t} \quad (3.2.18)$$

$$\Psi(y, t) = \Psi_0 + \varepsilon \Psi_1 e^{i\omega t} \quad (3.2.19)$$

Where $\varepsilon \ll 1$ is a parameter.

Equation 3.2.14 - 3.2.16 are reduced to

$$Du_0'' + ASu_0' - MU_0 = -BGr\theta_0 \quad (3.2.20)$$

$$DU_1'' + ASu_1' - (M + Ai\omega)u_1 = -BGr\theta_1 \quad (3.2.21)$$

$$E\theta_0'' + PrCS\theta_0' - Q\theta_0 = -Du\Psi_0'' - PrCQ_L\Psi_0 \quad (3.2.22)$$

$$E\theta_1'' + PrCS\theta_1' - (Q + PrCi\omega)\theta_1 = -Du\Psi_1'' - PrCQ_L\Psi_1 \quad (3.2.23)$$

$$\Psi_0'' + SSc\Psi_0' - KrSc\Psi_0 = 0 \quad (3.2.24)$$

$$\Psi_1'' + SSc\Psi_1' - (i\omega + Kr)Sc\Psi_1 = 0 \quad (3.2.25)$$

The boundary conditions are

$$u_0 = 1, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \Psi_0 = 1, \Psi_1 = 1 \text{ at } y = 0.$$

$$u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, \Psi_0 = 0, \Psi_1 = 0 \text{ at } y = \infty.$$

Equations (3.2.20)–(3.2.25) were solved and the solution for fluid velocity, temperature and the concentration was given by:

$$u(y, t) = (B_5 e^{-m_5 y} + B_3 e^{-m_3 y} + B_4 e^{-m_1 y}) + \varepsilon (B_8 e^{-m_6 y} + B_6 e^{-m_4 y} + B_7 e^{-m_2 y}) e^{i\omega t} \quad (3.2.26)$$

$$\theta(y, t) = (B_1 e^{-m_3 y} + A_1 e^{-m_1 y}) + \varepsilon (B_2 e^{-m_4 y} + A_2 e^{-m_2 y}) e^{i\omega t} \quad (3.2.27)$$

$$\Psi(y, t) = (e^{-m_1 y}) + \varepsilon (e^{-m_2 y}) e^{i\omega t} \quad (3.2.28)$$

Shearing stress

The dimensional form of the shearing stress at the plate is given by

$$\begin{aligned} \tau &= \left(\frac{\partial u}{\partial t} \right) \text{ at } y = 0 \\ &= (-B_5 m_5 - B_3 m_3 - B_4 m_1) + \varepsilon (B_8 m_6 - B_6 m_4 - B_7 m_2) e^{i\omega t} \quad (3.2.29) \end{aligned}$$

Heat Transfer Coefficient

The non-dimensional heat transfer coefficient in terms of Nusselt number is given by

$$\begin{aligned} Nu &= - \left(\frac{\partial \theta}{\partial t} \right) \text{ at } y = 0. \\ &= (B_1 m_3 + A_1 m_1) + \varepsilon (B_2 m_4 + A_2 m_2) e^{i\omega t} \quad (3.2.30) \end{aligned}$$

Sherwood Number

The non-dimensional mass transfer coefficient in terms of Sherwood number is given by

$$\begin{aligned} Sh &= - \left(\frac{\partial \Psi}{\partial t} \right) \text{ at } y = 0. \\ &= m_1 + \varepsilon m_2 e^{i\omega t} \quad (3.2.31) \end{aligned}$$

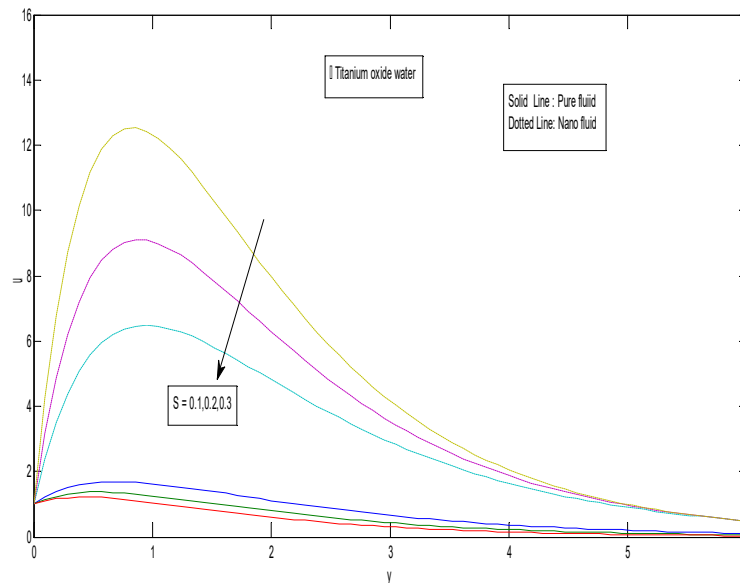


Figure 1. Velocity profile for Suction Parameter S

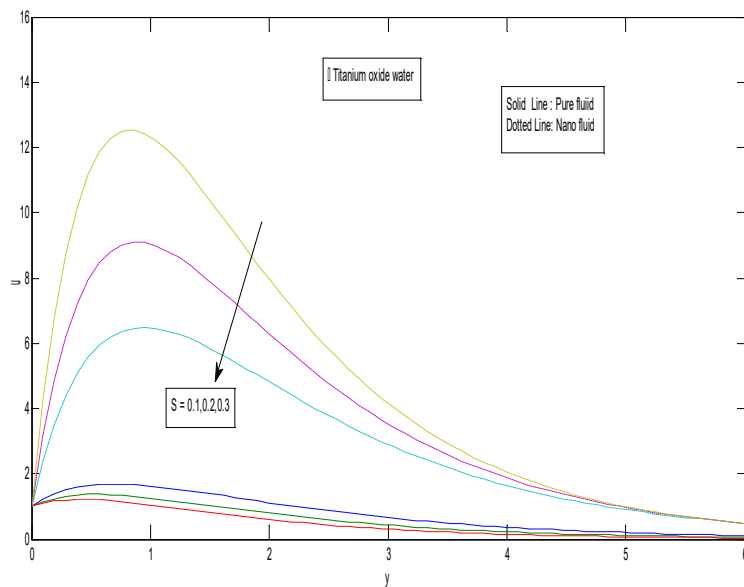


Figure 2 .Velocity Profile For Suction Parameter S

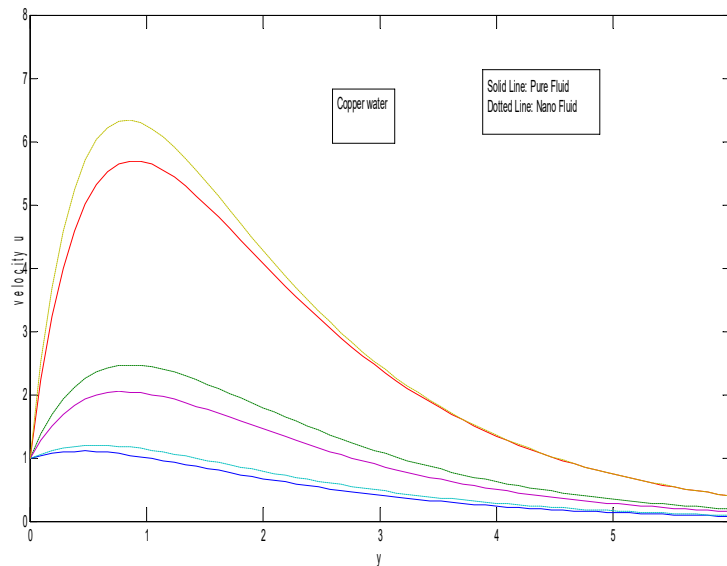


Figure 3. Velocity Profile For Radiation absorption parameter $Q_L = 1,2,3$.

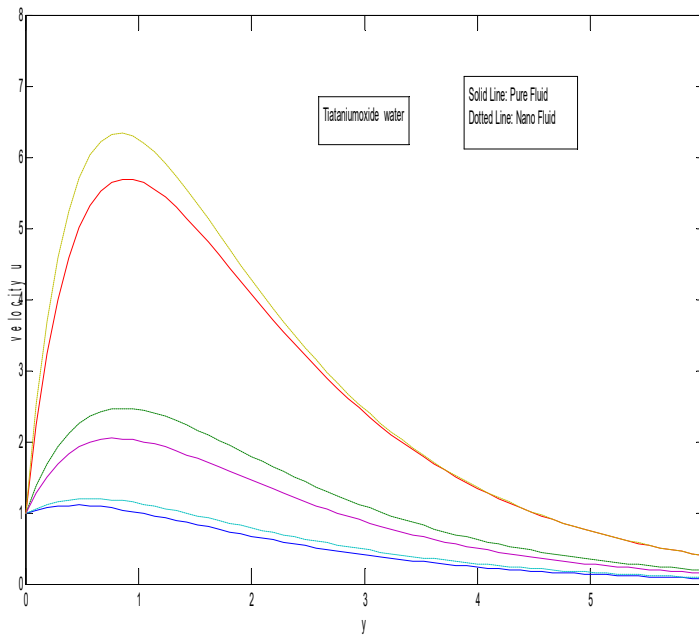


Figure 4. Velocity Profile For Radiation absorption parameter $Q_L = 1, 2, 3$.

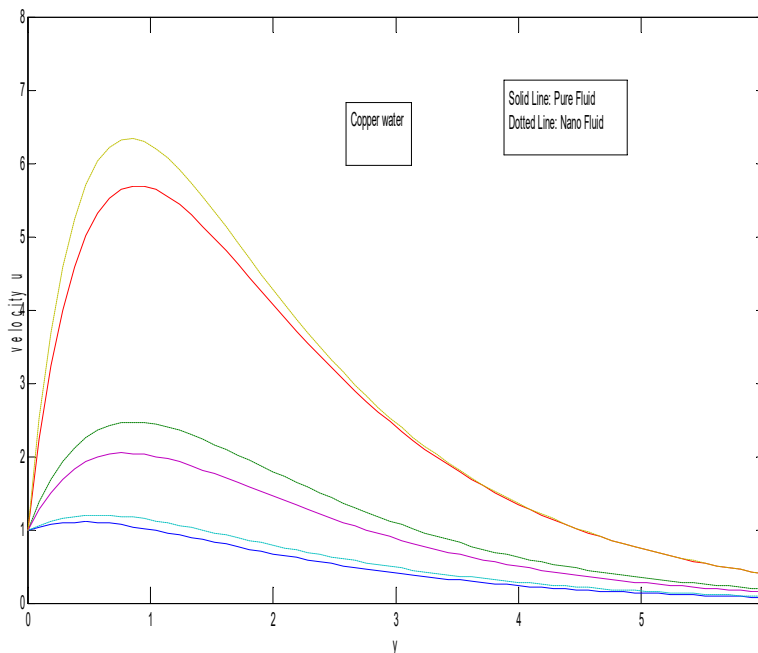


Figure 5. Velocity Profile For Magnetic field parameter $M = 0.2, 0.4, 0.6$

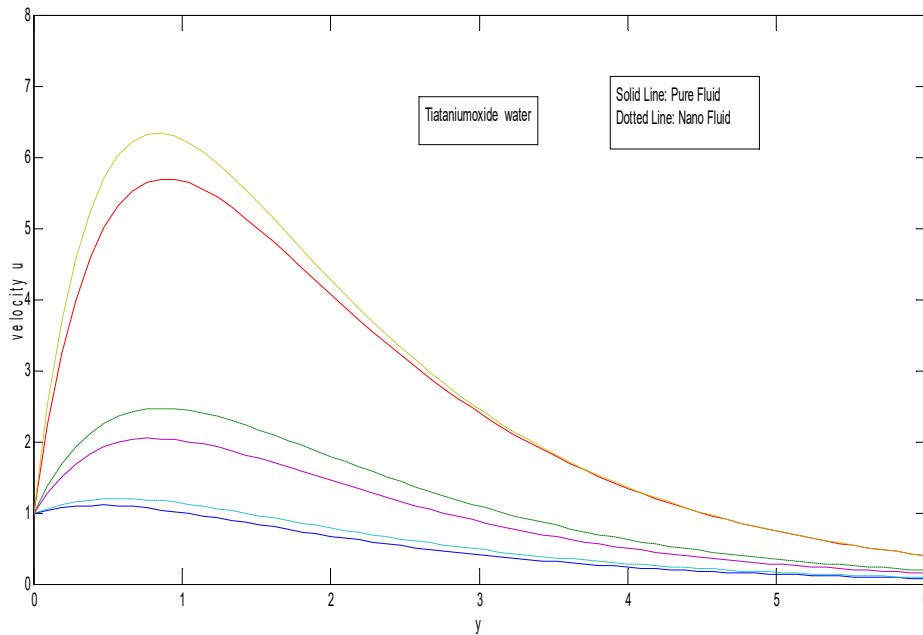


Figure 6. Velocity Profile For Magnetic field parameter $M= 0.2, 0.4, 0.6$

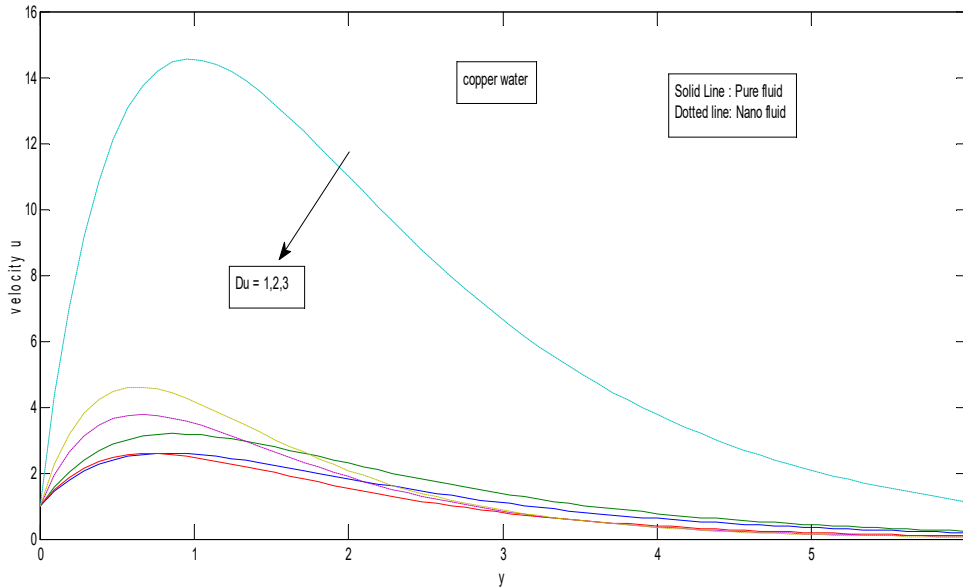


Figure 7: Velocity profiles For Dufour Number

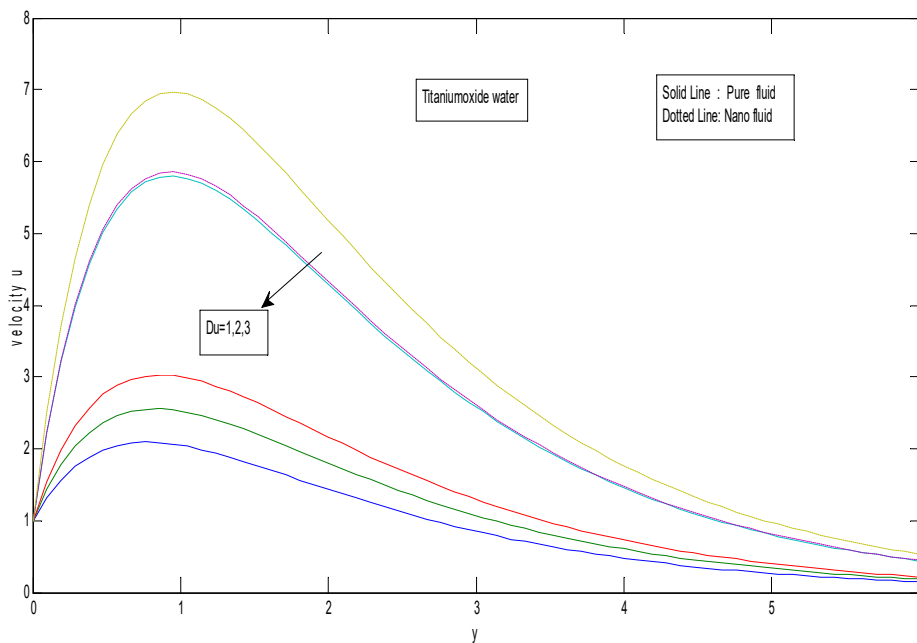


Figure 8: Velocity profiles For Dufour Number

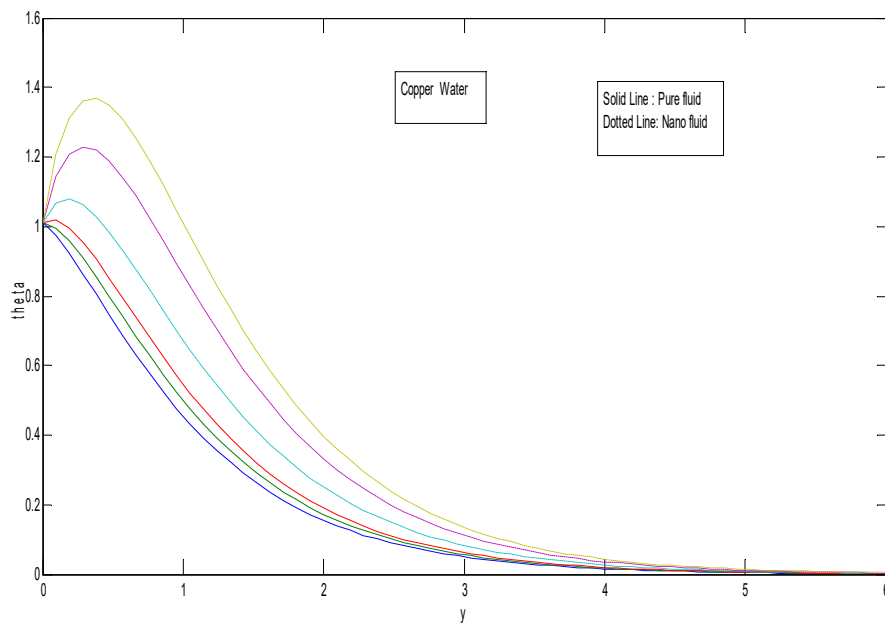


Figure 9: Temperature profiles for Radiation absorption Parameter $Q_L = 1, 2, 3$.

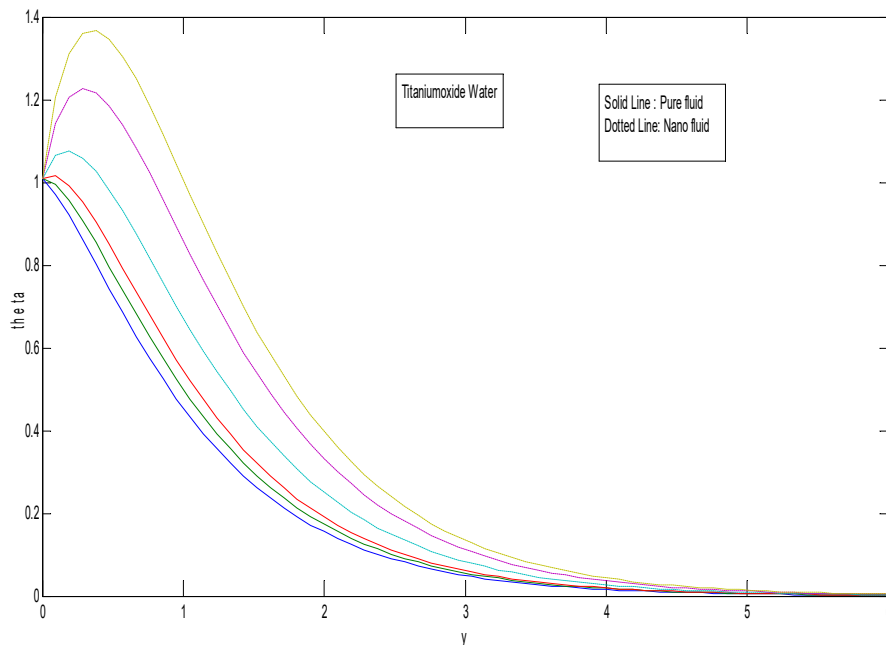


Figure 10: Temperature profiles for Radiation absorption Parameter $Q_L = 1,2,3$.

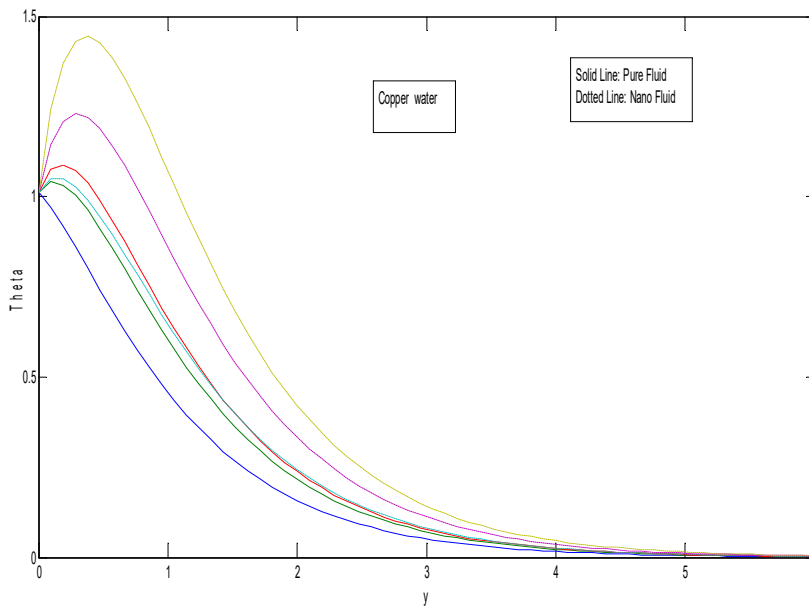


Figure 11. Temperature profile for Dufour number $Du = 1,2,3$.

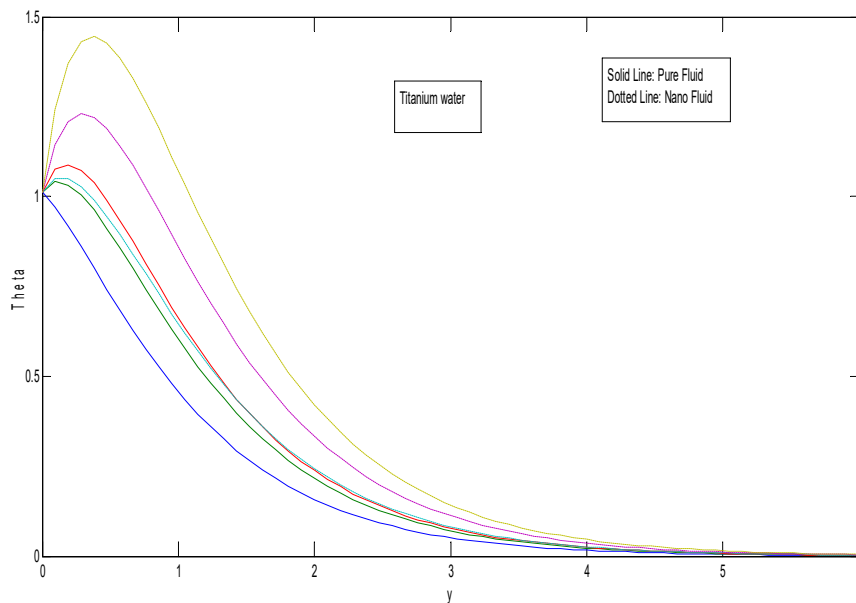


Figure 12. Temperature Profile for Dufour number $Du = 1, 2, 3$.

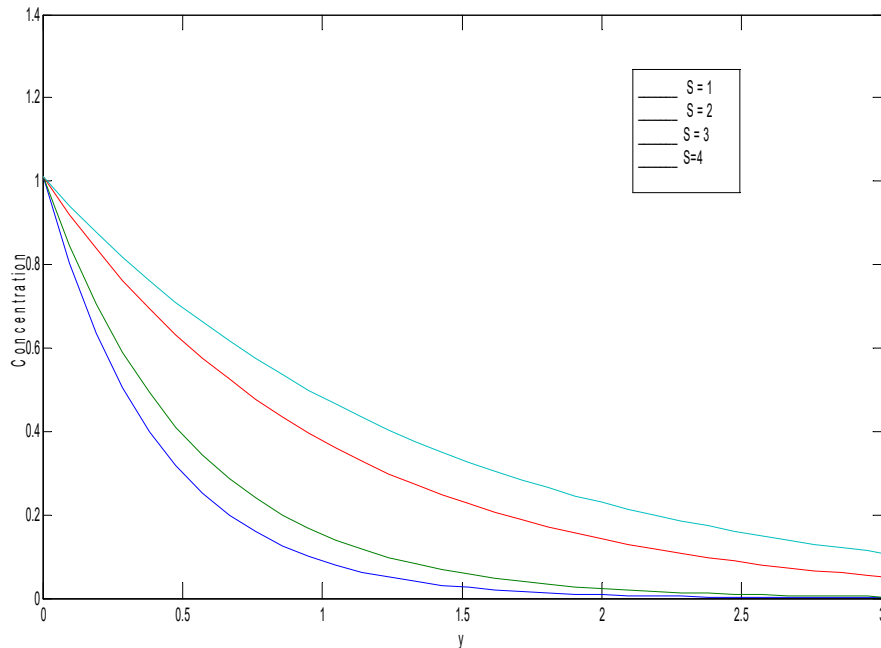


Figure 13. Concentration Profiles For Suction Parameter S .

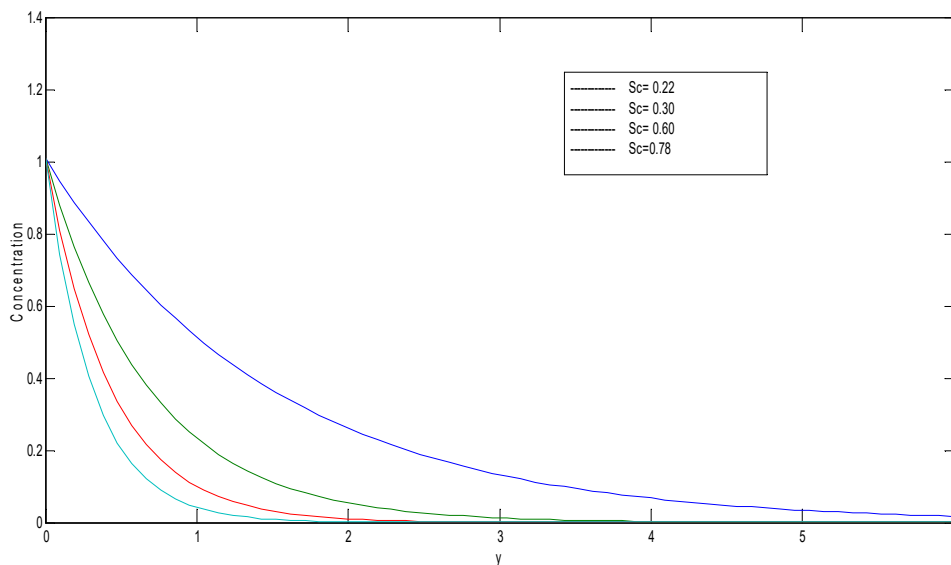


Figure 14. Concentration Profiles for Schmidt number Sc

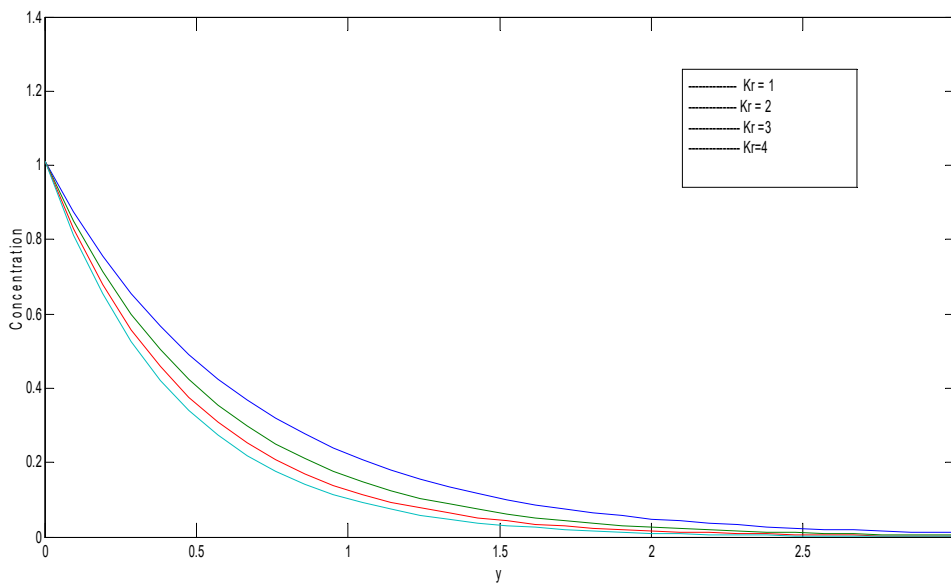


Figure 15: Concentration Profile for Suction Parameter S

IV. RESULTS AND DISCUSSION

In order to get physical insight into the problem, we have carried out numerical calculations for dimensionless variables for velocity field, temperature field, species concentration field, shearing stress, heat and mass transfer coefficient at the nanoparticles by

assigning some specific values to the parameters entering into the problem and the effects of these values are confirmed graphically. In this study, the values of the parameter τ , and Pr are taken fixed at 0.02, 1, 1, 0.71 respectively and the other parameters are chosen arbitrarily.

Figure 1 and 2 represents the effect of Suction parameter (S) on the velocity at a point in the fluid whereas Schmidt Number Sc , Chemical reaction parameter K_r , Heat source Q , Dufour number Du , Permeability Parameter K , Magnetic Parameter M , Q_L radiation absorption parameter, Grashof number Gr were taken as constant. From these figures, it was observed that velocity of the fluid across the boundary layer decreases with an increase in the suction parameter S for both regular fluid and nanofluid with Cu and TiO_2 nanoparticles

Figure 3 and 4 illustrates the effect of Radiation absorption parameter Q_L on velocity at any point in the fluid. From these graphs we arrived that the velocity profile increases with an increase in the radiation absorption parameter Q_L for both the regular fluid and nanofluid containing Cu and TiO_2 nanoparticles.

Figure 5 and 6 exhibits the effect of magnetic field, parameter M . It was observed that the velocity profile decreases with an increase in the strength of the magnetic field for both the base fluid and the nanofluid with Cu and TiO_2 nanoparticles.

Figure 7 and 8 demonstrates the effect of Dufour number Du . It indicates that the velocity profile increases with an increase in Dufour number for both the base fluid and the nanofluid with Cu and TiO_2 nanoparticles. It results in the boundary layer thickness.

Figure 9 and 10 represent the effect of the Radiation absorption parameter Q_L on temperature at any point in the fluid. From these graphs we arrived that the temperature increases with an increase in the radiation absorption parameter Q_L for both the regular fluid and nanofluid containing Cu and TiO_2 nanoparticles. It results in thermal boundary layer thickness. The nanofluid containing Cu –nanoparticles have thicker thermal boundary layer than the TiO_2 – nanoparticles.

Figure 11 and 12 depicts the effect of Dufour number Du . It indicates that the increasing values of Dufour number, the temperature was found to increase for both the base fluid and the nanofluid with Cu and TiO_2 nanoparticles. It results in the boundary layer thickness.

Figure 13 exhibit the effect of the Suction parameter S in the concentration profile. It shows that the concentration decreases with increasing the value of suction parameter (S).

Figure 14 exhibit the effect of the Schmidt number Sc in the concentration profile. It shows that the concentration decreases with increasing the value of suction parameter (S).

Figure 15 exhibit the effect of the Chemical reaction parameter K_r in the concentration profile. It shows that the increase in the value of chemical reaction parameter (K_r) will decrease the concentration for both the base fluid and the nanofluid with Cu and TiO_2 nanoparticles.

V. CONCLUSION

The two dimensional unsteady natural convective flow of a incompressible nanofluid past over a vertical permeable semi-infinite moving plate with constant heat source. The governing equations are solved analytically by using simple perturbation technique. The effects of different fluid flow parameter on velocity, temperature, the species concentration, rate of heat and mass transfer coefficient and the skin friction coefficient are derived and discussed through graphs.

The fluid velocity increases with an increasing value of Radiation absorption Q_L and the Dufour number Du and decreases with an increasing value of Suction (S) and magnetic field M for both the nanoparticles Cu and TiO_2 .

The temperature of the fluid increases with an increasing value of Dufour number Du and the radiation absorption Q_L for both the nanofluid containing Cu and TiO_2 containing nanoparticles. It results in the thermal boundary layer thickness.

The species concentration decreases with an increasing value of Chemical Reaction K_r , Suction Parameter S and the Schmidt number Sc .

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