

Review of Literature on Single and Multi-Objective Transportation Challenges

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Abstract: *This paper is a survey of finite number on different types of transportation problems discussed in literature to understand more about it. At present time, the transportation plays a crucial role in financial development, economic growth and civilization of the country as it enables trade between people. But for an effective transportation system and its perfect functioning, the problems arising from distribution of goods needs to be solved by means of methods discovered by various researchers, few of which are discussed in this paper using mathematical models, algorithms, figures and solved numericals. A general solution to the transportation can be found by optimizing a single or multi objective parameters such as cost of transportation, profit earned, total time of transportation etc. Transportation problems has a vast application in the field of operational research, economics, computer science, job scheduling and is used by many of the researchers for their research.*

Keywords: Deshpande's literary works

I. INTRODUCTION

In today's world of highly developing and competitive markets every institution providing different types of services requires a better and well developed transportation system for providing the various services to which involves proper scheduling of their products, delivery of their customers their products to the clients at minimum charges, with reliability of delivery, right quantity of the products to meet the demand of customers, in a minimum time and much more. Transportation includes in a major part of our lives which makes planning and operation of transportation a major issue. The problem of transportation is solved as a special class of Linear Programming Problem. Basically there are two types of transportation problems. First one is known as single objective transportation problem in which only a single variable can be optimized. And the second one is known as Multi objective transportation problems in which more than one variable is to be optimized. So the main purpose in the transportation problems is to get the optimal solutions. Optimization refers to best and efficient solution that can be obtain for a problem.

The basic step of solving a transportation problem is construction of balanced transportation table using the given information about demand and supply of the goods. When the total supply from the sources is equals to the total demand at the destination then the given transportation table is balanced and the table is not balanced if the two are not equal. We can make an unbalanced transportation problem balanced by inserting a duplicate row when requirement is greater than supply and by inserting a duplicate column when the requirement is less than total supply. The network representation of transportation problems is shown below:

Summary of Optimization Approaches

Ilija NIKOLIC (2006) proposed a method to solve transportation problems (i.e. the problems arising in distribution of goods produced by the sources to different markets/destinations) which is a very common problem. The objective of the paper is to reduce the overall charges and time of shipping of products or goods satisfying demand and supply requirements.

The given two forms of transportation time problems has been discussed in the paper.

Reduction of the overall time of shipping (which is taken as linear function), and is known as minimization of 1st transportation time.

reduction of the shipping time of the largest working shipping track (which is taken as non-linear function), and is known as minimization of 2nd transportation time.

Two algorithms are given in the paper for finding the solutions of both single objective and multi objective TPs. The procedure of solving single objective problem is explained as Algorithm 1 while the procedure of solving multi objective problem is explained as Algorithm 2. So, we can conclude that in this paper discusses the two objective problems (that is single and multi) and the preference is given to the total time of shipping. The solution for MOTP can be found by solving each objective of the problem independently as the same process that is used to solve a single objective problem.

Maurizio Bruglieri and Leo Liberti (2006) proposed a mathematical model for the common problems that arises from the planning and running of a biomass based energy production process such as quantity of biomass to purchase, decisions for transportation, designing of plant etc. The running model is taken as linear while the planning model is taken as non linear that can be figure out by converting it into a linear programming. There are two types of optimization problems that has been discussed in the paper. The first one is the modelling of production process explained using a typical process flowsheet (where the types of plants and end products are known). The another one is of deciding the types of plant to maximize the gain. The modelling process consists of crops as input, plants as intermediates and markets as output. The central concept for the model is taken as Process site. Detailed planning of process design and solution to that has also been discussed in the paper and lastly the reformulation for the problem to linearization technique. The proposed model is very advantageous if applied in the discussed way.

Veena Adlakha and Krzysztof Kowalski (2009) gave an effective method of solving transportation problems using shadow price theory and gave systematic observations to determine the optimal solution to the TP. The evolution of the Alternative Optimal Solutions (AOS) based on Shadow Price Theory is explained. For finding the optimal solutions firstly we determine the basic feasible solution which can be obtained by any of the methods like NWCR, Least Cost Method or VAM. After determining the basic feasible solutions, we obtain the dual prices for each supply constraint that is u_p and dual prices for each demand constraints that is v_q . If the cost cells are basic then the cost coefficient c_{pq} equals to $u_p + v_q$. We can solve the system of equations by putting any one of the u_p or v_q equals to zero. Then the net change in the total cost is found by the formula $c_{pq} - (u_p + v)$. Then the proposed method is used in working of the solution process. The values of $u_p + v_q$ are considered as the values of shadow prices and these are represented as shadow price matrix in the paper. The values of all shadow prices are smaller than the corresponding cost coefficients c_p . The method is step wise explained using the numerical problems in which the purpose is to determine the AOS (alternate optimal solution) and to find fixed load cells and cells with minimal load. On finding the optimal solution a post analysis has also done which can be very useful for the managers for assessing parallel solutions on behalf of the analysis of alternate load assignments.

Hardik Gangadwala and Dr. Jayesh Dhodiya (2012) developed a set of mathematical and statistical approaches that are effective in finding the precise result of single objective TP and multi objective TP with the help of Information and Communication Technology. The different solutions for enhancing the situation of roads has also been discussed in the paper. We will know about the amount of traffic present on the road. This helps the travellers/people to get an idea of how much it costs and how much time is required in travelling. Some of the tools and statistical and mathematical techniques that are listed in this paper are: Fake data detection tool, Predictive Mathematical Model and Numerical Technique, Road Situation, Developed Database and data warehousing equipment, Transportation Algorithm, Data mining techniques, Risk analysis tools, Internet Technology, Template etc. These statistical techniques will come out to be very profitable and useful for its users and operators if applied in proper way as discussed.

Hlayel Abdallah Ahmed (2012) developed a new solution method to solve optimization problem known as Best Candidates Method, where the purpose is to reduce total time of transportation. In other solution methods we find all the possible combinations and then compare them to determine the best solution while in some cases we directly approach the optimal solution but they do not give the best solution in 100 percent of the cases or some of the method are very time consuming. Whereas the BCM method chooses the best options to get the optimal result and reduces complexity of problem.

The proposed algorithm involves following steps:

STEP 1: Initially a transportation table is made for the problem and it is to be examine that the table is balanced but if the table is unbalanced, it is to be balanced by adding a duplicate row or column (but that duplicate row or column will not be consider in the solution process).

STEP 2: Choose the best candidate for the given problem. Choose best two candidates from each row and if contains same costs twice , elect that too.

STEP 3: Elect the best candidate from column also, if left but for the columns elect only one best candidates.

STEP 4: Now, choose the combinations and determine the best candidate for each row and column. Proceed similarly for all the rows and columns.

STEP 5: Compare the best candidates and add their respective costs to get the optimal solution.

The BCM method is then compared to Hungarian method to show that the proposed method is efficient and reduces complexity of the problem.

Mrs. Rekha Vivek Joshi(2013) proposed algorithm of four methods in which three variables can be optimized to decrease the total cost of transporting goods from one place to another with the help of numerical problems.

The first method that is discussed in the paper in *North West Corner Rule (NWCR)* and is used for obtaining initial basic feasible solutions for a TP. According to this rule, the maximum possible supply or demand units are allocated to the cell of top left corner of the transportation table and in the similar way process is proceeded further till the requirements of demand and supply are not completed.

The second method is *Least Cost Method*, in which allocation starts with the cell which has minimum value in the transportation table to obtain the initial basic feasible solutions. The same procedure is repeated until the rows and columns does not satisfy the demand and supply constraints as in North West Corner Rule.

The third one is *Vogel's Approximation Method* which is also used to obtain the basic feasible solution. In this method, the difference of minimum two costs are obtained from the respective rows and columns. Then a maximum difference among them is selected and the maximum demand/supply is allocated to the cell having minimum cost from that particular row or column. Similarly, the process moves further. This method gives optimal solution in about 80 percent of the cases.

Last one is *Modified Distribution Method (MODI)* which is a solution method for obtaining optimal solution to the TP. It is one of the standard methods that are known for obtaining optimal solutions. This method starts by obtaining basic feasible solution that can be determined by any of the above three discussed methods and further the problem is solved according to the proposed method.

Mohammed N.A.R, Lahji A.A. and Syed J.K (2013) presented a case study on shipping of eggs using Multi Objective Transportation Problem (MOTP). For maximizing profits it is necessary to transport goods from one place to another safely because perishable goods has no value, those items will be not taken into consideration and is therefore becomes loss for the producers or senders. The study is applied to the transportation of eggs as it is the daily need for most of the people in Andhra Pradesh from one city to another, in which the main motive was to deliver the packages with minimum breakages, minimum time, minimum distance and in lesser period of time. The whole process is classified into two sections: Surveying Phase and Mathematical Analysis. Surveying phase includes survey in various places of Andhra Pradesh and the collection of information from the whole sellers of that particular place. The mathematical Analysis includes the use of TORA software for this study to get optimal solution for minimizing the charges, time, distance and overall breakages of the eggs. Various aspects has been taken into consideration for the study such as speed limit because speedy drives will lead to more breakages of eggs and since broken eggs are direct loss for the seller. The proposed model is used to meet all the objectives which will be highly profitable for both the whole sellers as well as the suppliers. VAM is adopted for obtaining the initial basic feasible solution for given TPs.

Ram Vander Krogt, Leon Aronson, Nico Roos, Ces Witteveen and Jonne Zutt discussed about the tactical planning using heuristics techniques. They started by explaining about the tactical planner in detail and the possibilities of modification that can be done to it. Some initial analysis has also been performed (limited to static environment) and obviously the conclusion from those analysis. The main objective was to discover best planning strategies for adjusting the arrival of new orders which includes dealing with incidents. To heuristics are discussed in the paper. One is cooperative and thesecond one is competitive in detail using figures. The problems discussed in the paper is the

transportation problem that contains number of agents that can transfer a very large amount of products, (i.e. ship, truck etc.) basically, the paper works on an efficient strategy for planning the transportation problem tactical planners and operational planner transport agents are differentiated. The discussed technique is more efficient as compared to the initial one and is therefore fruitful for the development in business field.

Utpal Kanti Das, Md. Ashraful Babu, Aminur Rahmar and Dr. Md. Sharif (2014) obtained a computational error in initial VAM and proposed a developed Vogel's Approximation Method known as Advance Vogel's Approximation Method. In VAM, we obtain that difference of 2 least costs for all the rows as well as columns and then find the maximum difference and allocate the minimum demand/supply to the least value of that particular row or column. But sometimes we get two same least cost as result of which penalty becomes zero which creates a problem to practitioner which is resolved by this advanced method with a logical concept. Which is also very close to the optimal solution. Few numericals are solved using both the methods VAM and AVAM, and the result is compared for the better optimal solution. The advance method say if we have to equal cost in the cells of a row/column then we take the difference of one of that equal costs and the cost next to the smallest one. For example if the costs of a column are 4,4,6,5 then according to VAM penalty becomes zero but according to the new method, for obtaining penalty we will take the difference of 4 and 5 that is 1 and so the penalty came out to be 1. In this way, the computational error in the initial VAM is fixed.

Rabindranath Mondal, Farhan Rashid, Poly Rani Shaha and Raju Roy (2015) developed a modern technique to untangle TP with mixed constraints by giving a process method to determine an ideal MFL (More-For-Less). The ideal MFL solution is discussed in detail using solved numerical examples and flowchart in computer programming. Most of the real life transportation problems contains mixed constraints which can be easily sort out by this technique. The technique finds the initial basic feasible solution in a simple way that is easy to understand and use. There are so many methods to find initial basic solution for equality constraints but very few of the problems with mixed constraints are given in the research field due to the firmness essential in cracking these type of problems. According to this solution algorithm, as in VAM we find the difference of minimum two costs for each row and column. Then maximum supply and demand is allocated to the least element of row/column of maximum difference. A chart is also shown to explain the units that must be assigned to the demands and supply units. This paper proposed a method which is somehow similar to VAM and the numericals solved in the paper are then solved by simplex method and the result came out to be same from both the methods. Also, they used advanced computer program to solve these type of problems using simplex and the program is then examined whether the obtained programming is correct or not. The method is less time consuming and gives efficient results.

Qingyou Yan and Qian Zang(2015) proposed a model to solve MOTP along with the time window constraints (a condition which forces each vehicle to start with each customer at a period specified). Transportation problem is broadly studied in Logistics and Operation management where delivery of goods and products from the sources to targets is an essential concern. The proposed model is avail to obtain solution for MOTP in which there two objectives, first is to reduce the total charges of the transportation and the other is to minimize the size of number of vehicles used for the services. Formulation of the proposed model is elaborated and after that a genetic algorithm is modified to solve the problem for MOTP which is discussed stepwise. Also the conflicts

that are usually difficult to determine on solving with the classical methods can be easily determined by the given method. Various experiments has also been carried out using graphical representation to make comparison and getting the best optimal solutions. As a result of which they got the information about the trade off relation between the size of number of vehicles and total transportation cost. So, we can say that it came out to be very useful for the logistic enterprises to assure that the fleet size of the vehicles is convenient.

S.M. Abdul Kalam Azad, M.D. Bellel Hossain and Md. Mizanur Rahman (2017) approached a new method to find the solution for TPs based on total opportunity cost. The proposed algorithm deals with average of total cost of all the cells along each row considered as Row Average Total Opportunity Cost (RATOC) and mean of total cost of cell along each column considered as Column Average Total Opportunity Cost (CATOC). The given method is classified into two phases: Algorithm for total opportunity cost table (TOCT) and Algorithm for transportation allocation.

Mollah Mesbahuddin Ahmed, Aminur Rahman Khan, Md. Sharif Uddin, Faruque Ahmed (2016) proposed an advance method for finding the initial basic solution for a TP which is the basic step in solving for the best solution and the proposed process is known as Allocation Table Method (ATM). Several numericals are also solved using the proposed method which are explained in detail in the paper. The same numericals are then solved by some of the other famous methods to compare their results. The comparison is shown with 14 other methods and its very clear from the data that the proposed method gives the best optimal solution. This can be concluded that the proposed process is little more desirable than the other traditional algorithms known. So, it can be very beneficial for the organizations which wants to deliver their goods in an efficient cost to earn more profit. Given below is a flowchart to explain more about proposed algorithm:

Manmohan Maharana (2017) introduced a modern approach for determining the ideal solution for the TPs and that method is known as MM method. Few examples has also been illustrated to exemplify the method. A solution to a classical transportation problem generally consists of two parts. In the initial part, we obtain the initial basic feasible solution of the problem by applying some of very popular methods like NWCR, LCM or VAM and then the final step is of finding the optimal solutions using Modified Distribution or Stepping Stone (SS) Methods . But the proposed method for finding optimal solution needs basic calculations and help to get the solution directly in a single stage. The algorithm is very well explained step wise with the help of some solved numericals. It includes reduction of rows and columns step by step and by proceeding further in the similar way we get the solution to the problem. This method proves to be more efficient and is less time consuming. Also since its calculation is easy and contains less number of iterations. So, it proves to very simple to understand and apply.

Palanivel M. and Suganya M. (2018) developed a new mathematical technique to find the optimum solution for a transportation problem known as Harmonic Mean Method. Harmonic mean is the ratio of total number of observations to the sums of reciprocal of the number. The solution obtained from this method is somehow similar to the solution obtained by MODI Method and are closer to VAM Method. The method is explained using the numerical problems. In the proposed method we find the harmonic for all the respective rows and columns of balanced transportation table. The minimum supply or demand is then allocated to the least entry of the corresponding row or column of highest harmonic mean. The same procedure is repeated until all the supply and demand requirements met. And finally we determine the sum of product of the allocated supply and demand to their respective cost entries. The result is also compared to other known techniques of finding the basic feasible solution. The optimality of result is examined by using the MODI rule. By the comparison of results we can say that a global optimal solution can be found in less number of iterations.

II. CONCLUSION

This paper is a description of mathematical models, methods and algorithms for solving diverse types of TPs to get optimal solutions. For solving a transportation problems we need an effective an efficient method. By efficient method, we mean a method having less number of steps, is less time consuming, gives best optimal solution, easy to understand and advantageous for the users such as Harmonic Mean Method. For future work, we are looking forward to develop some more techniques for solving such problems and to find solution for the limitations of proposed methods.

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