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Some Identities for hv −Curvature Tensor in Generalized Recurrent Space

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Abstract: *The generalized $\mathcal{B}P$ −recurrent space introduced by [3]. Now, in this paper, certain identities belong to the mentioned space have been obtained.*

Keywords: Finsler space, generalized $\mathcal{B}P$ – recurrent space, hv −curvature tensor

I. INTRODUCTION

The generalized recurrence property has been studied by the Finslerian geometrics. Pandey et al. [12], Qasem and Abdallah [5] and Alaa et al. [4] introduced the generalized recurrent Finsler spaces for H^i_{jkh}, $R^i_{jkh}$and P^i_{jkh}, respectively. Also, the generalized property for normal projective curvature tensor N^i_{jkh} in sense of Berwald has been introduced by [6].Alaa et al. [1] studied certain identities in generalized $\mathcal{B}R$ – recurrent space. Further,Zlatanovic and Mincic [9] introduced several identities for some curvature tensors in generalized Finsler space.

Let F_n be an$n$ −dimensional Finsler space equipped with the metric function $F(x,y)$ satisfying the request conditions [7]. The vector y_i is defined by

(1.1) $y_i = g_{ij}(x,y)y^j$.

Two sets of quantities g_{ij} and its associative g^{ij} are connected by

(1.2) $g_{ij}g^{ik} = \delta^k_j = \begin{cases} 1 & if\ \ j = k\ , \\ 0 & if\ \ j \neq k\ . \end{cases}$

In view of (1.1) and (1.2), we have

(1.3) a) $\delta^i_k\, y^k = y^i$, b) $\delta^i_j g_{ir} = g_{jr}$, c) $\delta^i_k y_i = y_k$ and d) $\delta^i_i = n$.

Berwald covariant derivative $\mathcal{B}_k$ of an arbitrary tensor field T^i_j with respect to x^k is given by [2, 7]

$$\mathcal{B}_k T^i_j = \partial_k T^i_j - (\dot{\partial}_r T^i_j)G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}.$$

Berwald covariant derivative of the vector y^i and metric tensor g_{ij} satisfy

(1.4) a) $\mathcal{B}_k y^i = 0$ and b) $\mathcal{B}_k g_{ij} = -2C_{ijk|h}y^h = -2y^h\mathcal{B}_h C_{ijk}$.

The tensor P^i_{jkh}called hv −*curvature tensor* (*Cartan's second curvature tensor*) is positively homogeneous of degree -1 in y^i and defined by [7, 8]

$P^i_{jkh} = \dot{\partial}_h \Gamma^{*i}_{jk} + C^i_{jr}P^r_{kh} - C^i_{jh|k}$.

The associate tensor P_{ijkh}, torsion tensor P^i_{kh} and P −Ricci tensor P_{jk} of hv −curvature tensor P^i_{jkh} satisfies the relations

(1.5) a) $P_{ijkh} = g_{ir}\, P^r_{jkh}$, b) $P^i_{jkh}\, y^j = \Gamma^{*i}_{jkh}y^j = P^i_{kh} = C^i_{kh|r}y^r$ c) $P^i_{jki} = P_{jk}$

d) $P^i_{jik} = \bar{P}_{jk}$ and e) $P^i_{jk}\, y_i = 0$,

where $\bar{P}_{jk} = P_{jk} + S_{jk/o}$.

A Finsler spaces F_n which Cartan's second curvature tensor P^i_{jkh} satisfies the condition [3]

(1.6) $\mathcal{B}_m P^i_{jkh} = \lambda_m P^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh})$

called a *generalized* $\mathcal{B}P -$ *recurrentspace* which denoted it briefly by $G(\mathcal{B}P) - RF_n$.

The generalized $\mathcal{B}P$ −recurrent space which is P −reducible space will be called P −reducible generalized $\mathcal{B}P$ −recurrent space that denoted it briefly by P −reducible $-G(\mathcal{B}P) - RF_n$.

Transvecting the condition (2.1) by g_{il}, using (1.5a), (1.4b) and (1.3b), we get

(1.7) $\mathcal{B}_m P_{ljkh} = \lambda_m P_{ljkh} + \mu_m(g_{jl} g_{kh} - g_{kl} g_{jh}) + 2P^i_{jkh} y^t \mathcal{B}_t C_{ilm}$.

II. MAIN RESULTS

In this section, we obtained some identities in generalized $\mathcal{B}P$ −recurrent space. Let us consider $G(\mathcal{B}P) - RF_n$.

We know the projective curvature tensor P^i_{jkh} satisfying the following [10]

(2.1) $\lambda_m P^i_{jkh} + \lambda_k P^i_{jhm} + \lambda_h P^i_{jmk} = 0$.

From the condition (1.6), we conclude

$$\mathcal{B}_m P^i_{jkh} + \mathcal{B}_k P^i_{jhm} + \mathcal{B}_h P^i_{jmk} = \lambda_m P^i_{jkh} + \lambda_k P^i_{jhm} + \lambda_h P^i_{jmk} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) + \mu_k(\delta^i_j g_{hm} - \delta^i_h g_{jm}) + \mu_h(\delta^i_j g_{mk} - \delta^i_m g_{jk}).$$

Using (2.1) in above equation, we get

(2.2) $\mathcal{B}_m P^i_{jkh} + \mathcal{B}_k P^i_{jhm} + \mathcal{B}_h P^i_{jmk} = \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) + \mu_k(\delta^i_j g_{hm} - \delta^i_h g_{jm}) + \mu_h(\delta^i_j g_{mk} - \delta^i_m g_{jk})$.

Transvecting (2.2) by y^j , using (1.5b), (1.4a), (1.3a) and (1.1), we get

(2.3) $\mathcal{B}_m P^i_{kh} + \mathcal{B}_k P^i_{hm} + \mathcal{B}_h P^i_{mk} = \mu_m(y^i g_{kh} - \delta^i_k y_h) + \mu_k(y^i g_{hm} - \delta^i_h y_m) + \mu_h(y^i g_{mk} - \delta^i_m y_k)$.

Contracting the indices i and h in (2.2), using (1.5c), (1.5d), (1.3b) and (1.3d), then using the symmetric property of metric tensor g_{jk}, we get

(2.4) $\mathcal{B}_m P_{jk} + \mathcal{B}_k \bar{P}_{jm} + \mathcal{B}_i P^i_{jmk} = \mu_k(1 - n) g_{jm} + \mu_i(\delta^i_j g_{mk} - \delta^i_m g_{jk})$.

Transvecting (2.3) by y_i , using (1.5e), (1.3c) and ($y_i\ y^i = F^2$), we get

(2.5) $\mu_m(F^2 g_{kh} - y_k y_h) + \mu_k(F^2 g_{hm} - y_h y_m) + \mu_h(F^2 g_{mk} - y_m y_k) = 0$.

Thus, we conclude

Corollary 2. 1. *In $G(\mathcal{B}P) - RF_n$, we have the identities (2.2), (2.3), (2.4) and (2.5).*

In P −reducible space, we have the following identity [11]

(2.6) $P_{ijkh} + P_{jhki} + P_{hikj} = 0$

Taking $\mathcal{B}$ −covariant derivative for the left side of (2.6) with respect to x^m, we get

$\mathcal{B}_m P_{ijkh} + \mathcal{B}_m P_{jhki} + \mathcal{B}_m P_{hikj} = 0$.

Using (1.7) in above equation, we get

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$$\lambda_m(P_{ijkh}+P_{jhki}+P_{hikj})+2(P^l_{jkh}+P^l_{jhk}+P^l_{hkj})\mathcal{B}_m g_{li}$$
$$+\mu_m(g_{ij}g_{kh}-g_{ik}g_{jh}+g_{jh}g_{ki}-g_{jk}g_{hi}+g_{hi}g_{kj}-g_{hk}g_{ij})=0.$$

Using (2.6) and (1.4b) in above equation and using the symmetric property of metric tensor g_{jh}, we get

(2.7) $\quad (P^l_{jkh}+P^l_{jhk}+P^l_{hkj})y^t\mathcal{B}_t C_{ijk}=0.$

Thus, we conclude

Corollary 2. 2.*In* P −reducible $-G(\mathcal{B}P)-RF_n$. *we have the identity (2.7).*

III. CONCLUSION

Two corollaries related to generalized$\mathcal{B}P$ −recurrent space have been obtained and proved. Assured identities belong to it have been concluded.

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