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Analytical and Dynamic analysis of a Prey-Predator Model

Tripesh Kumar Tiwari[1], Dhirendra Kumar Pandey[2], Amit Kumar Yadav[3]

Assistant Professor, Department of Applied Science

Babu Sundar Singh Institute of Technology and Management, Lucknow (AKTU), UP[1]

Assistant Professor, Department of Applied Science, Allenhouse Institute of Technology, Rooma, Kanpur[2]

Assistant Professor, Department of Mechanical Engineering

Babu Sundar Singh Institute of Technology and Management, Lucknow[3]

Abstract: *Mathematical and dynamic analysis of the Prey-Predator model in the presence of Altenative prey with impulsive state feedback control , and Predator -Prey process extended their application and have given rise to system which represents more real different bilogical issues that appear in the context of interacting spicies .Our aim in this paper isto give a state of the art review of recent Predator-Prey models which include some more interesting properties such that Allee effect ,fear effect immigration etc.We generate the qualitative results obtained for each of them.*

Keywords: Mathematical and dynamic analysis

I. INTRODUCTION

Predator-Prey models such as the Lotka -volterra model can be used in a homogeneous environment.However ,more generally,the environment is heterogeneous and this can be represented using a set of discrete patches connected by migrations.Prey -Predator models with infected prey have already been analysedextensively.seeBairragi et al.(2007);Ghosh et al .(2007); Bhattacharya and Bhattacharya (2006);Xiao and chen (2001);Chattopadhyay and Atino (1999);Venturino (1995,1994).Very recently also more complex situations like the infection of Predators through the consumption of prey .see Hsieh and Hsiao (2008) and the references therein ,as well as the influence of prey infection on the chaotic dynamics of a three - trophic food chain.see Das et al.(2009),were considered .

This paper is organized as follows: First we present the complete model.then we show that by use of aggregation methods ,it is possible to build a global Predator –Prey model governing the total Prey and Predator densities ,by total predator density we signify the Predator density obtained by summation over all individual Predator categories such as searching ,handling hawk and dove sub-populations.there after ,we present the result of the bifurcation analysis of the aggregated methods with respect to two relevant parameters,the carrying capacity and the costs for fight .The article ends with a general discussion on advantages of different tactics and their efferts on the stability of the Predator-Prey system.

Predator –prey model is autonomous model corresponding to the model was first proposed by Maynard Smith which accounts for the allelopathic interaction between two competing species .Preliminary stability analysis of the model with respect to the context of Phytoplankton allelopathy was proposed by Chattopadhyay and in here ,the allelopathic interaction can strengthen the stability of coexistence steady –state is shown .In reality ,the introduction of allelopathic interaction into the competition model can result in more complicated dynamics including the presence of two co-existing steady –state and their stability depends upon the magnitudes of the parameters .Interestingly,

The proposed model can exhibit the bi-stable scenario for a range of parameter values and the coexistence of the competing species depend upon the initial population densities .A through analysis of the model proposed by Maynard - Smith is recently investigated by Gupta et al with a few Modifications for a different context .Dynamics of model with discrete time delay and almost periodic coefficients are investigated by Abbas et al .In this work ,the role of time delay and environmental quasi-periodicity on the bloom formation by two competing phytoplankton species are throughly investigated .

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Several authors have considered the non –autonomous models of interacting species , which is capable to take care of seasonal variation in environmental

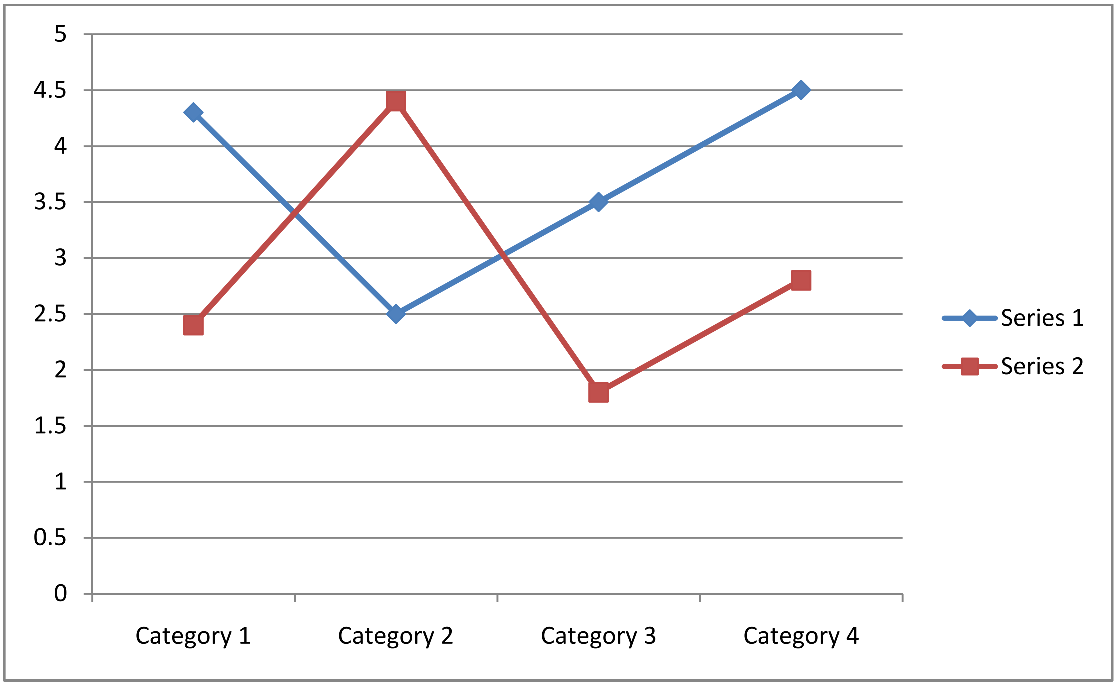
Conditions .The con- cerned models are of Lotka-Volterra type but without allelopathic interaction term various facters (intencity of sun light ,temperature ,salinity of water ,amount of soluble nutri-entsetc) in the environment varies seasonally ,in a periodic manner ,and affect the intrinsic parameters involved the mathematical models of interacting populations .A part from these ,the assumption behind periodic oscillation of the system parameters are justified due to other seasonal factors like mating habites ,availlability of food etc .hence its quite reason –able to study the non –autonomous models of ecological system driven by periodic external forces .Many researchers have studied the non –autonomous ecological models where intrinsic rates are considered as bounded periodic functions of time to model the seasonal variabil –ity .Motivated by this fact ,in this work we consider the deterministic model where the parameters of the system are periodic with common period ,the second assumption is due to the simplicity of mathematical calculations .

The main problem in the study of populations growth model with periodic coefficients is the existence of the positive periodic solution and its global stability .Hence its is reasonable to search for conditions under with the concerned non –autonomous system with periodic coefficients with have a positive periodic solution which is globally asymptotically stable .in this context ,we assume that the parameters in the system are periodic in ‘ t’ with a fixed period t>0 and with derive the parametric restriction in terms of the bounds of the periodic coefficients for the existence of positive periodic solutions and its global asymptopic stability for the model .

Numericals bifurcation analysis

Predator –prey model going on non-linear system ,consider a forest where only foxes and rabbits are presents .Foxes eat rabbits and rabbits eat clover which is sufficient in the forest and the rabbits never run out of food .When the rabbits are abundant ,then the foxes flourish and their population increases .So they eat too many rabbits and then the rabbits and their populations decreases .As the rabbit population decreases ,there is a luck of food for foxes and hence their population begins to decline .As the population of foxes decreases ,the rabbits become relatively safe and then their population starts to increase again .In this way, we have an endlessly repeated cycles of interrelated increases and decreases in the populations of rabbit and fox ,which can be understood by the following graph :

Fig a:



Series -1=Rabbits (x)

Series-2=Foxes (y)

In from figure ,abovezig-zag lines intersect each other and these are the all possible points when fox meet with rabbit.

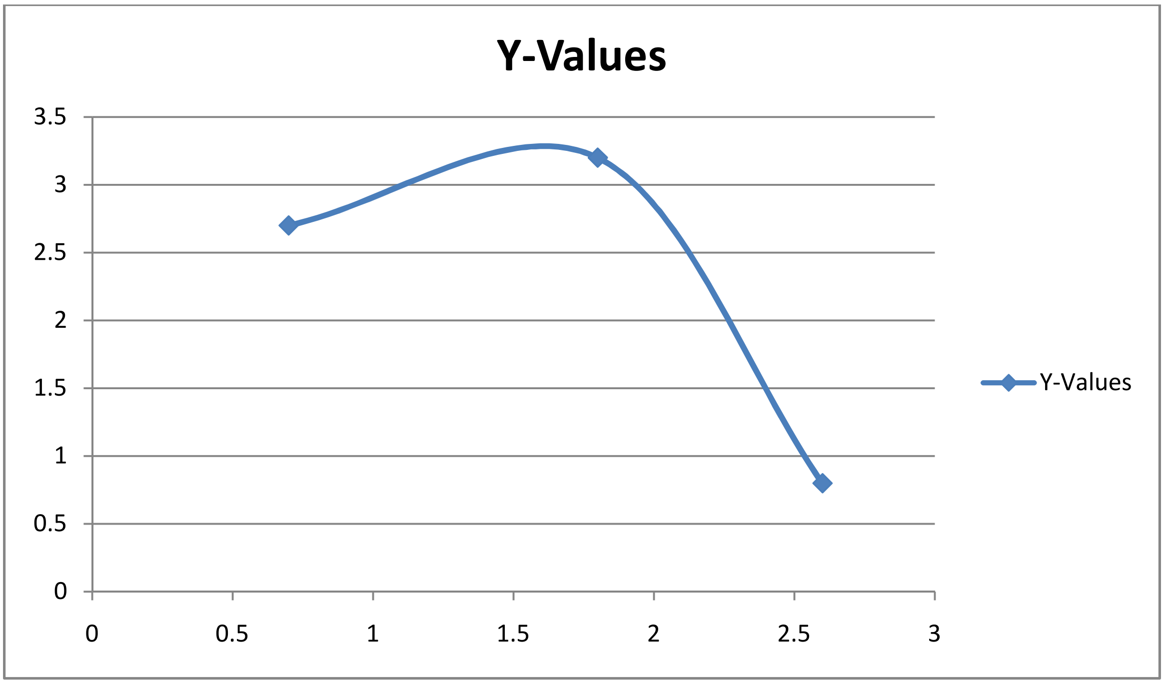


Fig b: Stable and unstable behavior of rabbit and Fox.

In Fig:b Vertical line Express the behavior of Fox and horizontal line expressed the behavior of rabbits.Here Stability regions y- show separated by a straight line with slop 2D and meets the abscissa after some time Graphical representation of above criterion .

Now if we seenFig(a) ' t'is the time and $\ddot{x}, \ddot{y}$ are population of rabbits and foxes at any time 't' respectively .

If x is the number of rabbits at any time' t'and if there are no foxes , then the relation

dx/dt =ax, a> 0 .

should hold .this relation states that the rate of increase of number of rabbits is proportional to the number of rabbits present at that time .

In a similar way ,if y is the number of foxes at any time 't' then

dy/dt = -cy , c > 0.

(Here negative sign is due to the fact that increases of population of fox results decreases of population of rabbit).

Let the number of encounters between rabbits and foxes per unit of time is proportional to xy .if we further assume that a certain proportion of those encounters results in a rabbit being eaten, then we have

dx /dt =ax-bxy, a,b> 0 .

Similarly ,in the absence of rabbits, the fox population decreases and their increase depends on the number of encounters with rabbits ,then we have

dy/dt =-cy +dxy . c,d>0.

In this way we have following non-linear system of equations as :

dx/dt =x(a-by) ....(i)

dy /dt =-y(c-dx) ....(ii)

The equations in system (i)and (ii) Predator –prey equations .

Example (i):Now to solve above both equations ,eliminate' t ' in equations (i) and (ii) and then separate the variables ,i-e,

Fig.(C)A bloom event is triggered by suppressing the initial zooplankton Concentration (middle Panel)sufficiently ,far below the stationary value z_s (red

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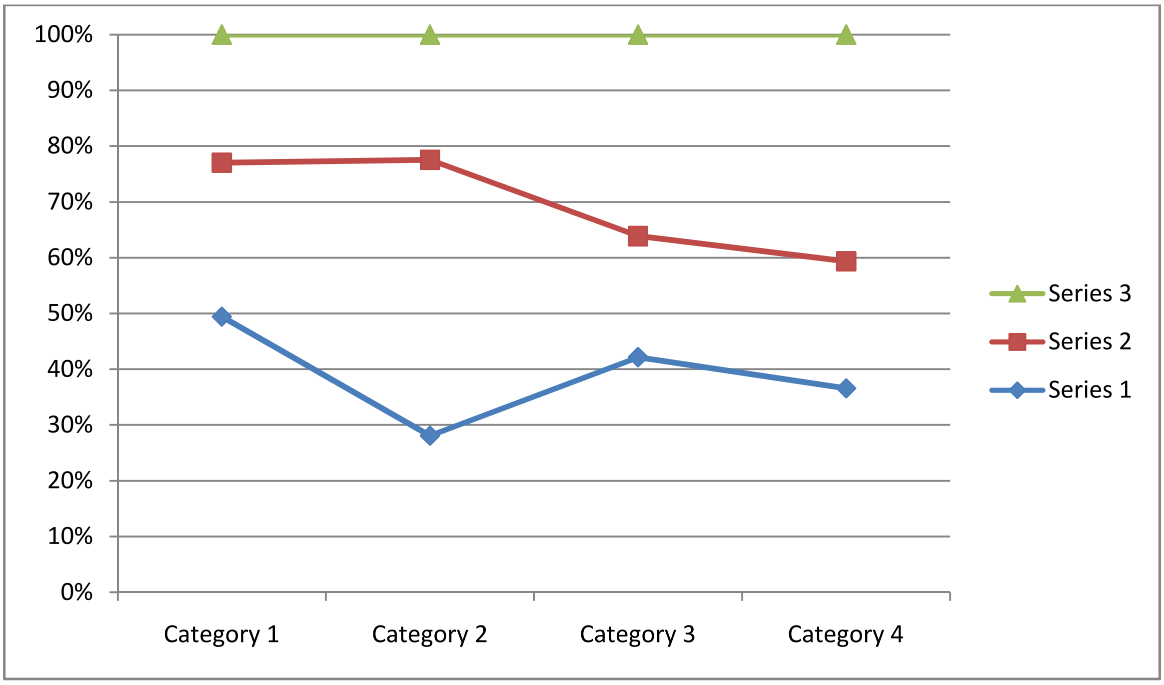
curved line).The initial phytoplankton concentration (Top panel)is fixed in the stationary value p_s(red curved line).In the bottom panel the event is illustrated in (P,Z) state space with relevant nullclines plotted using red curved lines.



dx /x(a-by) =dt

dy/-y(c-dx)=dt

hence dx/x(a-by)=dy/-y(c-dx)

(a-by)dy/y=- (c-dx)dx/x

$\left[\frac{a}{y} - b\right] dy = -\left|\frac{c}{x} - d\right|$dx

On integrating, we get

a logy –by =-c logx +dx +logC

or$y^a e^{-by} = C_1 x^{-c} .e^{dx}$.....................................(2)

if we take x=x_0 ,y= y_0 at t=0, then from equation (2)

$C_{1=} x_0^c y_0^a e^{-dx_0-by_0}$...(3)

For convenient ,let$C_1 = K$,then equation (2) takes the form

$$y^a e^{-by} = Kx^{-c} e^{dx}$$

Which is the solution of the system (1)where K=c_1 is given by (3).

Example-2.Formation non –linear second order equation satisfied by the function x(t) ,with help of predator equations . Since predator equations are given by

dx/dt =x(a-by) ....(a)

dy/dt =-y(c-dx) ....(b)

from (a) ,we have

$\frac{d^2x}{dt^2}$ = dx/dt(a-by)-bxdy/dt

=dx/dt($\frac{1}{x}\frac{dx}{dt}$)-bx $\{-y(c-dx)\}$

=$\frac{1}{x}(\frac{dx}{dt})^2$+ bxy (c-dx)

=$\frac{1}{x}(\frac{dx}{dt})^2$ +bx(c-dx) $\left\{\frac{1}{b}\left(a-\frac{1}{x}\frac{dx}{dt}\right)\right\}$

=$\frac{1}{x}(\frac{dx}{dt})^2$+ax (c-dx)-(c-dx)$\frac{dx}{dt}$

Or x$\frac{d^2x}{dt^2}$ – $(\frac{dx}{dt})^2$+x(c-dx)$\frac{dx}{dt}$ ax^2(c-dx)=0.

II. DISCUSSION AND RESULT

As for as observability of chaos in nature is concerned it can be divided into two distinct categories (i)Robust chaos ,and (ii)Short-term recurrent chaos .We give precise definitions for both of them and present results which suggest that robust chaos should be rarely found in nature .

(i)Robust chaos :If chaotic dynamics exists in a reason 2 D parameter space spanned by two crucial parameters of a model systems and also if the basin of attraction for the associated attractor is large ; one understands that the chaotic dynamics displayed **by the system is Ro**bust .If this is the case ,chaos would be dominant dynamical mode of the corresponding real system .

(ii)Short-term recurrent chaos :There exist two mechanism which generate this kind of chaotic behaviour .If chaotic dynamics is display region in2D parameter space of a model system .It can still support short-term recurrent chaos(strc)provided the form of chaotic behaviour ,where in is interrupted by other kind of dynamic at irregular and unpredictable intervals. Therefore we see that short –term recurrent chaos can be caused either by deterministic changes in the system parameters or by exogenous stochastic influences .In the former ,chaotic dynamic is interrupted by smooth changes in system's parameters where as stochastic influences affect such interruptions in the para one .

III. CONCLUSIONS

Thus **it** is clear that robust chaos is less likly to be found in nature .instead ,what nature seems to abound in has resently been called strc. The strc can occur through to mechanism .The first mechanism involve deterministic changes in the system parameters. The other invokes exclusive role of abrupt change in initial condition .The only systems ,where in robust chaos has been observed so far ,are diffusively coupled predator –prey systems.Diffusion couples stable limit cycles oscillators on a special gradient .These couples oscillators force each other at ''incommensurate '' frequencies to generate chaos.The chaotic dynamics exist in a region of non zero measure 2D parameters space and there is no other competing attractor in the initial condition space. Thus ,there is no intermixing of the attractor basins .on the contrary ,those with generalist top –predator are governed by deterministic change in system parameters we opine that the natural system with first kind of food chain would present difficult challengers as for as program of quantification of their dynamical complexity is concerned .The other kind of systems seems to allow such as program to be implemented smoothly .This conjecture is to be tested in the laboratory and in the field .

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