IJARSCT

ISSN (Online) 2581-9429

International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)

International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal

Impact Factor: 7.301

Volume 3, Issue 2, August 2023

Hamiltonian Cycle and Hamiltonian Path and its Applications-A Review

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Abstract: *Hamiltonian cycle and Hamiltonian path are fundamental graph theory concepts that have significant implications in various real-world applications. This paper provides an overview of these concepts, their characteristics, and the practical domains where they find valuable applications*

Keywords: Hamiltonian cycle, Hamiltonian path

I. INTRODUCTION

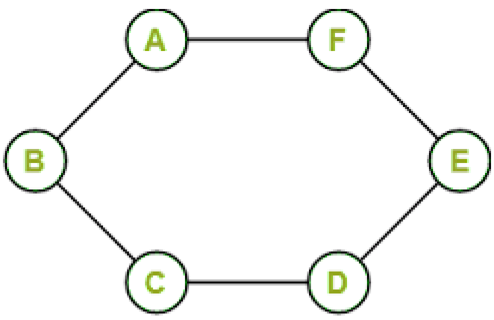
Graph theory is a fundamental branch of mathematics that studies the relationships between objects represented as vertices, connected by edges. Two intriguing and practical concepts in graph theory are the Hamiltonian cycle and Hamiltonian path. These concepts have significant applications in various real-world scenarios, making them essential and valuable tools in diverse fields. The significance of Hamiltonian cycles and paths lies in their ability to solve practical problems in different domains. One prominent application is in transportation and logistics. Identifying Hamiltonian cycles in transportation networks, such as road networks or airline routes, allows for the optimization of delivery routes, reducing travel time and costs. Moreover, these cycles ensure efficient resource utilization and enhance the overall performance of transportation systems.

1.1 Hamiltonian Cycle:

A Hamiltonian cycle, also known as a Hamiltonian circuit, is a concept in graph theory that refers to a closed path in an undirected graph that visits each vertex exactly once and returns to its starting vertex. In other words, it is a cycle in the graph that passes through every vertex exactly once and comes back to the starting vertex, forming a closed loop[1]

Formally, given an undirected graph G with vertices V and edges E, a Hamiltonian cycle is a sequence of vertices $(v_1, v_2, v_3 \ldots\ldots v_n, v_1)$ such that each vertex in V appears exactly once in the Sequence and for each consecutive pair of vertices (v_i, v_{i+1}) there is an edge (v_i, v_{i+1}) in E. Additionally, there must be an edge (v_n, v_1) to complete the cycle.

Ex:



Hamiltonian cycle $A \to B \to C \to D \to E \to F \to A$

1.2 Hamiltonian Path

A Hamiltonian path is a concept in graph theory that refers to an open path in an undirected graph, which visits each vertex exactly once, without necessarily forming a closed loop. Unlike the Hamiltonian cycle, the Hamiltonian path does not require the path to return to the starting vertex[2]

DOI: 10.48175/IJARSCT-12766

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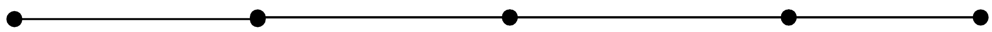
Formally, given an undirected graph G with vertices V and edges E, a Hamiltonian path is a sequence of vertices $(v_1, v_2, v_3 ......v_n, v_1)$ such that each vertex in V appears exactly once in the sequence, and for each consecutive pair of vertices (v_i, v_{i+1}) there is an edge (v_i, v_{i+1}) in E.

Ex: A B C D E



Hamiltonian path

The following resultsare based on Hamiltonian cycle and Hamiltonian path :

Theorem 1: For each odd integer, $n \geq 5$ such that $n-1 = 1 \bmod 4, J_n \; is \; K^{+1}$ -Hypo-edge-Hamiltonian-$t^* - laceable$ where $1 \leq t \leq diamJ_n$ [4]

Theorem 2: The flower snark J_n, if for every odd positive integer, $n \geq 7$ such that $n-1 = 1 \bmod 4$, then $J_n \; is \; K^{+1}$ -Hypo-edge-Hamiltonian-$t^* - laceable$ where $1 \leq t \leq diamJ_n$ [4]

Theorem:3 For every positive integer $n \geq 2$, J_{2n} is K^{+0} Hypo-edge-hamiltonian $t^* - laceable$ For t=1 and K^{+1} Hypoedge-hamiltonian for $2 \leq t \leq diamJ_{2n}$ [5]

Theorem 4: Let P_m and P_n be two paths. If m and n are odd integers such that $(m, n) \geq 5$ and ($m \geq n$) then the Cartesian product $P_m \, X \, P_n$ is K^{+1} hypo edge-Hamiltonian- t^*-laceablefor odd t ($1 \leq t \leq DiamG$)[5]

Theorem 5: Let P_m and P_n be two paths. If m and n are even integers such that $(m, n) \geq 4$ and ($m \geq n$) then the Cartesian product $P_m \, X \, P_n$ is K^{+1} hypo edge-Hamiltonian- t^*-laceablefor even t ($1 \leq t \leq DiamG$)[5].

1.3 Applications

Hamiltonian cycles and Hamiltonian paths, while challenging to find in large graphs, have numerous practical applications across various fields. Some of the key applications include:

1. **Transportation and Logistics:** In transportation networks, finding Hamiltonian cycles can optimize delivery routes for goods and services, minimizing travel time and costs. These cycles ensure that each location is visited exactly once, leading to efficient resource utilization and improved logistics[7,8].
2. **Computer Networks:** Hamiltonian paths play a crucial role in designing optimal data transmission routes between nodes in computer networks. By determining the shortest path that visits each node exactly once, network latency can be reduced, resulting in faster data transfer and enhanced network performance[9,10].
3. **Circuit Design and Chip Testing:** Hamiltonian cycles and paths are utilized in circuit design and chip testing to verify the correctness of connections and detect faults in integrated circuits. Ensuring the presence of a Hamiltonian cycle in a circuit guarantees the reliability and proper functioning of electronic devices.[11,12].
4. **Bioinformatics and Molecular Chemistry:** In bioinformatics, Hamiltonian cycles and paths are applied to model protein folding patterns, DNA sequencing, and drug discovery. Identifying Hamiltonian paths in protein structures aids in understanding their functions and interactions, contributing to drug development and disease research[13,14].

5. **Robotics and Path Planning:** In robotics, Hamiltonian paths are employed to plan efficient and collision-free paths for robots navigating through complex environments. The path should cover all necessary locations without revisiting any vertex, ensuring that the robot reaches its destination optimally[15,16].
6. **Game Theory and Tournaments:** Hamiltonian cycles have interesting applications in game theory, specifically in tournament design. In the context of round-robin tournaments, Hamiltonian cycles determine a fair schedule where each participant competes against every other participant exactly once[17 - 19].
7. **DNA Sequencing and Genome Assembly:** Hamiltonian cycles and paths are useful in DNA sequencing, where they aid in reconstructing the full genome from smaller fragments, ensuring that no segment is repeated, and every portion is covered[20 - 23].
8. **Urban Planning:** In urban planning, Hamiltonian cycles can be utilized to optimize city infrastructure, designing efficient routes for public transportation, and connecting key locations in a city[24,25].

While finding Hamiltonian cycles and paths in large graphs is computationally challenging, their applications in solving real-world optimization problems make them essential tools in various scientific, engineering, and logistical domains. Researchers continue to explore innovative algorithms and heuristics to tackle these problems efficiently and further extend their practical applications.

II. CONCLUSION

In conclusion, Hamiltonian cycles and Hamiltonian paths are having broad practical applications. These concepts find valuable applications in transportation, computer networks, circuit design, bioinformatics, robotics, game theory, DNA sequencing, urban planning, and more. They enable efficient route optimization, enhance network performance, verify circuit connections, aid in protein folding prediction, plan collision-free robot paths, design fair tournaments, and optimize urban infrastructure.

Despite their NP-completeness, researchers continue to explore innovative algorithms and heuristics to efficiently tackle these challenging problems. As technology advances, the significance of Hamiltonian cycles and paths is expected to grow, unlocking new possibilities for optimization and resource management in various domains. Their diverse applications make them indispensable tools in solving real-world challenges and advancing scientific, engineering, and logistical endeavors.

III. ACKNOWLEDGMENTS

All authors thankful to Management and the staff of the Mathematics department, R L J I T, Doddaballapur, Bengaluru, for their support and encouragement.

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