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Einstein's Generalities and their Solutions for Different Combinations of Scalar, Massive Gravitational and Electromagnetic Fields

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Abstract: *In this present paper we have obtained the of Einstein's Field Equations for the plane wave solutions* $\sqrt{y^2+z^2}-t$ *with one time exist. In general theory of relativity in the four different combinations as follows:*
(a) Zero massive scalar field coupledwith gravitational field.
(b) Zero mass scalar field coupledwith gravitational &electromagnetic field.
(c) Massive scalar field coupled withgravitational field.
(d) Massive scalar field coupled withgravitational &electromagnetic field

Keywords: Einstein's field equation, Mathematical formulation of EFEs, Scalar fields, gravitational fields, electromagnetic fields, the energy momentum tensor & the fundamental tenser.

I. INTRODUCTION

The Einstein's equations (EFEs) in the GR corresponds to the geometry of spacetime-to the distribution of matter with in it. EFEs are published in 1915 in the form of tenser equations which related the local spacetime curvature with the local energy, momentum & stress within the spacetime as a stress tensor. Similar to this the electromagnetic fields are related to the distribution of charges of currents via Maxwell's equations. The relation between Einstein tensor & the metric tensor gives that the EFEs expressed as the set of nonlinear partial differential equations when used like this. The components of metric tenser are the solutions of EFEs. The inertial trajectories of particles & radiation ie geodesics in the geometry are then calculated by considering the geodesics equation.

The EFEs can be reduced to Newton's law of gravitation in the limit of weak by using the local energy-momentum tensor. The exact solutions for the of EFE are obtained by simplifying the assumptions like symmetry. Some special classes of exact solutions are studied because they many gravitational phenomena like rotating black holes of the expanding universe. In approximating the spacetime the additional simplification is brought out as having only small deviations from flat spacetime giving to the linearized EFE which are used to study the gravitational waves.

The Mathematical form of the EFEs is given by

$$G_{\mu\nu} + \Lambda_{g\mu\nu} + k\,T_{\mu\nu}$$

Where, $G_{\mu\nu}$ denotes the metric tenser,
$\Lambda_{g\mu\nu}$ denotes the Einstein tenser,
$k\,T_{\mu\nu}$ denotes the stress-energy tensor
Λ is the cosmological constant & k is the Einstein gravitational constant.
Also $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R_{g\mu\nu}$$

where, $R_{\mu\nu}$ the Ricci creative tensor
Also, $k = \frac{8\pi G}{C^4} \approx 2.076647442844 \times 10^{-4}$
G in the Newtonian constant of gravitation

C is the speed of light in vacuum.

Hence the above EFE is expressed a

$$R_{\mu\nu} - \frac{1}{2} R_{g\mu\nu} + \Lambda_{g\mu\nu} = k\, T_{\mu\nu}$$

II. PRELIMINARIES

The non-flat plane wave solutions of g_{ij} of the field equations $R_{ij} = 0$ has obtained by Takeno H [2] & established the existence $(z - t)$ & $\left(\frac{t}{z}\right)$-type plane waves for purely gravitational case in four dimensional empty regions, of space time. Similar to this in paper [1] Kadhao & Thengane have obtained the plane wave solutions of $R_{ij} = 0$ in V_4 in two time axes. In the present paper we have established the existence of $\sqrt{y^2 + z^2} - t$ -type plane waves in V_4 with one time axis by investigating the line element, investigating the line element,

$$ds^2 = -Adx^2 + \frac{1}{y^2 + z^2}[(-A + B)\, y^2 dy^2 + (-A + B)z^2 z^2 + 2yzBdydz] + \frac{2}{y^2 + z^2}[-y(A + B)dydt - zBdzdt] + (A + B)dt^2 \tag{2:1}$$

We have obtained the relation of nonvanishing components of Ricci tensor as

$$P = \left(\frac{y^2 + z^2}{y^2}\right) R_{22} = \left(\frac{y^2 + z^2}{yz}\right) R_{23} = \left(-\frac{\sqrt{y^2 + z^2}}{y}\right) R_{24} = \left(\frac{y^2 + z^2}{y^2}\right) R_{33} = \left(-\frac{\sqrt{y^2 + z^2}}{z}\right) R_{34} = R_{44}$$
$$= \frac{1}{2A}\left(\bar{\bar{A}} - \frac{\bar{A}^2}{2A} - \frac{\bar{A}\bar{B}}{B}\right) + \frac{1}{2B}\left(\bar{\bar{B}} - \frac{3\bar{A}^2}{2B}\right) \tag{2.2}$$

for $[\sqrt{y^2 + z^2} - t\,]$ −type plane wave.

In this paper we investigate whether this types of plane wave solution exist in the case where zero mass scalar field coupled with gravitational field and the zero mass scalar field coupled with gravitational and electromagnetic field. Furthermore, we consider the coupling of massive scalar field with gravitational field and the massive scalar field with gravitational and electromagnetic field also in V_4 to investigate the existence of these this type of plane wave solutions EFEs.

$$R_{ij} = (-8\pi)\left[T_{ij} - \tfrac{1}{2} g_{ij}\, T\right], \quad (i.j = 1,2,3,4) \; (2.3)$$

where R_{ij} is the Ricci tensor of the space time,

T_{ij} is the energy momentum tensor,

g_{ij} is the fundamental tensor of the space time, and $T = T_i^i = g^{ij}\ T_{ij}$

III. $[\sqrt{y^2 + z^2} - t\,]$ −TYPE PLANE WAVE SOLUTIONS.

Zero mass scalar field coupled with gravitational field

The is energy momentum tensor of zero mass scalar field given by

$$T_{ij} = \tfrac{1}{4\pi}\left[V_i V_j - \tfrac{1}{2} g_{ij}\ V_k V^k\right] \quad (k = 1,2,3,4) \; (3.1)$$

where V is scalar function of Z and $V_j = \frac{dv}{dx^j}$, $(x^j = x, y, z, t)$

Thus,

$$\left(\tfrac{\sqrt{y^2+z^2}}{y}\right) V_2 = \left(\tfrac{\sqrt{y^2+z^2}}{z}\right) V_3 = -V_4 = \bar{V} \quad since\ V_1 = 0 \; (3.2)$$

where $(-)$ bar denotes partial derivative w.r.t. z.

From the line element (2.1) we have,

$$g_{22} = \left(\frac{y^2}{y^2 + z^2}\right)(-A + B), \quad g_{23} = \left(\frac{y^z}{y^2 + z^2}\right) B, \quad g_{24} = \left(\frac{-y}{\sqrt{y^2 + z^2}}\right)(A + B)$$

$$g_{33} = \left(\frac{z^2}{y^2+z^2}\right)(-A+B), \quad g_{34} = \left(\frac{-z}{\sqrt{y^2+z^2}}\right)B, \quad g_{44} = A+B$$

$$g^{22} = -\left(\frac{y^2+z^2}{y^2}\right)\frac{1}{2A}, \quad g^{24} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)\frac{1}{2A}, \quad g^{33} = -\left(\frac{y^2+z^2}{z^2}\right)\left(\frac{A+B}{A^2}\right) \tag{3.3}$$

$$g^{34} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)\frac{1}{2A}, \quad g^{44} = \left(\frac{A-2B}{2A^2}\right) \tag{3.4}$$

$\Rightarrow V_k \ V^k = 0$ (3.5)

Therefore equation (3.1) become

$$T_{ij} = \frac{1}{4\pi}\left[V_i \ V_j\right] \ (3.6)$$

Then from equation (3.6) we have,

$$T = T_i^i = g^{ij} \ T_{ij} = 0 \ (3.7)$$

Then using the equation (3.6) and (3.7) the EFEs (2.3) becomes,

$$R_{ij} = -2\left[V_i \ V_j\right] \ (3.8)$$

which further gives,

$$R_{22} = \left(\frac{-2y^2}{y^2+z^2}\right)\bar{V}^2, \quad R_{33} = \left(\frac{-2z^2}{y^2+z^2}\right)\bar{V}^2, \quad R_{44} = -2\,\bar{V}^2, \ R_{23} = \left(\frac{-2yz}{y^2+z^2}\right)\bar{V}^2,$$

$$R_{24} = \left(\frac{2y}{\sqrt{y^2+z^2}}\right)\bar{V}^2, \quad R_{34} = \left(\frac{2z}{\sqrt{y^2+z^2}}\right)\bar{V}^2, \tag{3.9}$$

$$\Rightarrow \left(\frac{y^2+z^2}{y^2}\right)R_{22} = \left(\frac{y^2+z^2}{z^2}\right)R_{33} = R_{44} = \left(\frac{y^2+z^2}{y^2}\right)R_{23} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)$$

$$= R_{24} = \left(\frac{-\sqrt{y^2+z^2}}{z}\right) R_{34} = P \ \ (3.10)$$

It is observed that equation (3.10) is compatible with the equation (2.2) which is obtained in the case of purely gravitational field.

Hence, we have,

$[\sqrt{y^2+z^2} - t \ ]$ −type plane wave solution exists in the case where zero mass scalar field is coupled with the gravitational field.

IV. ZERO MASS SCALAR FILED COUPLED WITH GRAVITATIONAL & ELECTROMAGNETIC FILED

In this combination the EEFs (2.3) gives

$R_{ij} = -2\left[V_i \, V_j + 4\pi E_{ij}\right]$ which gives,

$$R_{22} = -2\left[\frac{y^2\,\bar{V}^2}{y^2+z^2} + 4\pi E_{22}\right], \quad R_{23} = -2\left[\frac{y^z\bar{V}^2}{y^2+z^2} + 4\pi E_{23}\right]$$

$$R_{24} = -2\left[\frac{-y^2\,\bar{V}^2}{\sqrt{y^2+z^2}} + 4\pi E_{24}\right], \quad R_{23} = -2\left[\frac{z^2\bar{V}^2}{y^2+z^2} + 4\pi E_{33}\right]$$

$$R_{34} = -2\left[\frac{-z\,\bar{V}^2}{\sqrt{y^2+z^2}} + 4\pi E_{34}\right], \quad R_{44} = -2[\bar{V}^2 + 4\pi E_{44}] \tag{4.1}$$

which is incompatible with the equation (2.2), so the solution does not exist.

V. MASSIVE SCALAR FILED COUPLED WITH GRAVITATIONAL FILED

The energy momentum function tensor of massive scalar filed is given by,

$$T_{ij} = \frac{1}{4\pi}\left[V_i\, V_j - \frac{1}{2} g_{ij}(V_k\, V^k - m^2 v^2)\right],\ k = 1,2,3,4 \ldots \ (5.1)$$

where V is a scalar function of Z and $V_j = \frac{\partial V}{\partial x^j}$, $(x^j = x, y, a, t)$

m is mass associated with the massive scalar field.

Thus,

$$\left(\frac{\sqrt{y^2+z^2}}{y}\right) V_2 = \left(\frac{\sqrt{y^2+z^2}}{z}\right) V_3 = V_4 = \bar{V} \qquad \therefore V_1 = 0 \ (5.2)$$

where $(-)$ bar denotes partial derivative w.r.t. z from the line element (2.1) we have,

$$g_{22} = \left(\frac{y^2}{y^2+z^2}\right)(-A+B), \quad g_{23} = \left(\frac{y^z}{y^2+z^2}\right)B, \quad g_{24} = \left(\frac{-y}{\sqrt{y^2+z^2}}\right)(A+B)$$

$$g_{33} = \left(\frac{z^2}{y^2+z^2}\right)(-A+B), \quad g_{34} = \left(\frac{-z}{\sqrt{y^2+z^2}}\right)B, \quad g_{44} = (A+B) \qquad (5.3)$$

$$g^{22} = -\left(\frac{y^2+z^2}{y^2}\right)\frac{1}{2A}, \quad g^{24} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)\frac{1}{2A}, \quad g^{33} = -\left(\frac{y^2+z^2}{z^2}\right)\left(\frac{A+B}{A^2}\right)$$

$$g^{34} = -\left(\frac{-\sqrt{y^2+z^2}}{z}\right)\frac{B}{A^2}, \quad g^{34} = \left(\frac{A-2B}{2A^2}\right) \qquad (5.4)$$

$$\Rightarrow V_k\, V^k = 0 \ (5.5)$$

Therefore, the equation (4.1) implies

$$T_{ij} = \frac{1}{4\pi}\left[V_i\, V_j - \frac{1}{2} g_{ij}\, m^2 v^2\right] (5.6)$$

Equation (4.6) yields

$$T = T_i^i = g^{ij}\ T_{ij} = \frac{1}{2}\, m^2 v^2 \ (5.7)$$

Using (5.6) and (5.7) EFEs (2.2) becomes,

$$R_{ij} = -2\left[V_i\, V_j - \frac{1}{2} g_{ij}\, m^2 v^2\right] (5.8)$$

which future yields,

$$R_{22} = -2\left[\left(\frac{y^2}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{y^2}{y^2+z^2}\right) m^2 v^2\right]$$

$$R_{33} = -2\left[\left(\frac{z^2}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{z^2}{y^2+z^2}\right)(-A+B) m^2 v^2\right]$$

$$R_{44} = -2\left[\bar{V}^2 - \frac{1}{2}(A+B) m^2 v^2\right]$$

$$R_{23} = -2\left[\left(\frac{y^z}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{y^2}{y^2+z^2}\right) B m^2 v^2\right]$$

$$R_{24} = -2\left[\left(\frac{-y}{\sqrt{y^2+z^2}}\right)\bar{V}^2 + \frac{1}{2}\left(\frac{y}{\sqrt{y^2+z^2}}\right)(A+B) m^2 v^2\right]$$

$$R_{34} = -2\left[\left(\frac{-z}{\sqrt{y^2+z^2}}\right)\bar{V}^2 + \frac{1}{2}\left(\frac{z}{\sqrt{y^2+z^2}}\right) B m^2 v^2\right] (5.9)$$

But from the line element (2.1) we have the relation of nonvanishing components of Ricci tensor as

$$\left(\frac{y^2+z^2}{y^2}\right)R_{22} = \left(\frac{y^2+z^2}{yz}\right)R_{23} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)R_{24}\ \left(\frac{y^2+z^2}{z^2}\right)R_{33} = \left(\frac{-\sqrt{y^2+z^2}}{z}\right)R_{34} = R_{44} \qquad (5.10)$$

It is to be noted that here the equation (5.9) is incompatible with the equation (5.10) which is obtained in the case of purely gravitational field. Hence we have,

$[\sqrt{y^2+z^2}-t\ ]-$ type plane wave solutions does not exist in the case where massive scalar field is coupled with gravitational field.

Remark: $If\ m^2 = 0$ them the equation (5.9) is compatible to (5.10) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational field.

VI. MASSIVE SCALAR FIELD COUPLED WITH GRAVITATIONAL & ELECTROMAGNETIC FIELD.

In this case the energy momentum tensor is given by,

$$T_{ij} = \frac{1}{4\pi}\left[V_i\,V_j - \frac{1}{2}g_{ij}\,(V_kV^k - m^2v^2) + E_{ij}\right] \quad (6.1)$$

where E_{ij} denotes the electromagnetic energy momentum tensor.

But as in the previous case $V_kV^k = 0$

Therefore equation (6.1) becomes

$$T_{ij} = \frac{1}{4\pi}\left[V_i\,V_j - \frac{1}{2}g_{ij}\,m^2v^2\right] + E_{ij} \quad (6.2)$$

Also (6.2) implies,

$$T = T_i^i = g^{ij}T_{ij} = \frac{1}{2\pi}\,m^2v^2 \quad (6.3)$$

Hence Einstein's field equations (2.3) becomes,

$$R_{ij} = -2\left[V_i\,V_j - \frac{1}{2}g_{ij}\,m^2v^2 + 4\pi E_{ij}\right] \quad (6.4)$$

Then from (6.4) we have

$$R_{22} = -2\left[\left(\frac{y^2}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{y^2}{y^2+z^2}\right)(-A+B)\,m^2v^2 + 4\pi E_{22}\right]$$

$$R_{33} = -2\left[\left(\frac{z^2}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{z^2}{y^2+z^2}\right)(-A+B)m^2v^2 + 4\pi E_{33}\right]$$

$$R_{44} = -2\left[\bar{V}^2 - \frac{1}{2}(A+B)m^2v^2 + 4\pi E_{44}\right]$$

$$R_{23} = -2\left[\left(\frac{y^z}{y^2+z^2}\right)\bar{V}^2 - \frac{1}{2}\left(\frac{yz}{y^2+z^2}\right)Bm^2v^2 + 4\pi E_{23}\right]$$

$$R_{24} = -2\left[\left(\frac{-y}{\sqrt{y^2+z^2}}\right)\bar{V}^2 + \frac{1}{2}\left(\frac{y}{\sqrt{y^2+z^2}}\right)(A+B)m^2v^2 + 4\pi E_{24}\right]$$

$$R_{34} = -2\left[\left(\frac{-z}{\sqrt{y^2+z^2}}\right)\bar{V}^2 + \frac{1}{2}\left(\frac{z}{\sqrt{y^2+z^2}}\right)Bm^2v^2 + 4\pi E_{34}\right] \quad (6.5)$$

But the line element (2.1) gives the relation of non-vanishing components of Ricci tensor as

$$\left(\frac{y^2+z^2}{y^2}\right)R_{22} = \left(\frac{y^2+z^2}{z^2}\right)R_{33} = R_{44} = \left(\frac{y^2+z^2}{y^2}\right)R_{23} = \left(\frac{-\sqrt{y^2+z^2}}{y}\right)R_{24} = \left(\frac{-\sqrt{y^2+z^2}}{z}\right)R_{34} \quad (6.6)$$

Here it has been observed that the equation (6.5) is incompatible with the equation (6.6) which is obtained in the case of purely gravitational field. Hence we have,

$\sqrt{y^2+z^2}-t$ -type plane wave solutions of Einstein's field equation in general relativity doesn't exist in the case where massive scalar field is coupled with gravitational and electromagnetic field.

VII. CONCLUSION

The existence of plane wave solutions are given by (3.10), (4.1), (5.10), & (6.6) in four different cases.

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