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Observations on Homogeneous Bi-Quadratic Equation with Five Unknowns $(x^4 - y^4) + 2(x-y)(x^3+y^3) = 36(z^2 - w^2)p^2$

J. Shanthi[1] and M. A. Gopalan[2]

Assistant Professor, Department of Mathematics[1]

Professor, Department of Mathematics

Shrimati Indira Gandhi College, Trichy, Tamil Nadu, India

Affiliated to Bharathidasan University, Trichy,Tamil Nadu,India

shanthivishvaa@gmail.com [1]and mayilgopalan@gmail.com[2]

Abstract: In this paper ,we present non-zero integer solutions to homogeneous quinary bi- quadratic equation $(x^4 - y^4) + 2(x-y)(x^3+y^3) = 36(z^2 - w^2)p^2$

Keywords: homogeneous bi-quadratic ,quinary bi-quadratic, integer solutions

I. INTRODUCTION

It is worth to observe that higher degree Diophantine equations with multiple variables are rich in variety. In this context , one may refer [1-31] for various problems on biquadratic equations with three ,four and five variables. . While attempting to collect homogeneous bi-quadratic Diophantine equations with five unknowns ,the authors came across the paper [32] represented by

"Observations On Homogeneous Bi-quadratic Equation with five unknowns $(x-y)+2(x-y)(x^3+y^3)=36(z^2-w^2)p^2$ ". In the above paper ,the authors have presented a few choices of integer solutions. Albeit tacitly , there are other choices of integer solutions to the considered equation which is the main aim of this paper.

1.1 Method of Analysis

The homogeneous bi-quadratic diophantine equation with five unknowns under consideration is

$$(x^4 - y^4) + 2(x-y)(x^3+y^3) = 36(z^2 - w^2)p^2 \quad (1)$$

Introduction of the linear transformations

$$x = 6(u+2v), y = 6(u-2v), z = 6(uv+2), w = 6(uv-2), p = 2q \quad (2)$$

in (1) leads to

$$u^2 + 8v^2 = q^2 \quad (3)$$

which can be solved through different methods. In view of (2), different sets of integer solutions to (1) are obtained.

Set 1:

It is observed that (3) is satisfied by

$$v = 2rs, u = 8r^2 - s^2, q = 8r^2 + s^2$$

In view of (2) ,the integer solutions to (1) are given by

$$x = 6(8r^2 - s^2 + 4rs), y = 6(8r^2 - s^2 - 4rs),$$

$$z = 6[2rs(8r^2 - s^2) + 2], w = 6[2rs(8r^2 - s^2) - 2], p = 2(8r^2 + s^2)$$

Set 2:

Express (3) as the system of double equations as below in Table 1:

Table 1-System of double equations

System	I	II	III	IV	V
$q+u$	v^2	$2v^2$	$4v^2$	$8v$	$4v$
$q-u$	8	4	2	v	$2v$

Solving each of the above system of equations ,the values of u, v, q are obtained.In view of (2) ,the corresponding integer solutions are obtained. For simplicity ,the solutions are exhibited below:

Solutions from System I :

$$x = 6(2s^2 + 4s - 4), y = 6(2s^2 - 4s - 4),$$

$$z = 6(2s(2s^2 - 4) + 2), w = 6(2s(2s^2 - 4) - 2), p = 2(2s^2 + 4)$$

Solutions from System II :

$$x = 6(s^2 + 2s - 2), y = 6(s^2 - 2s - 2),$$

$$z = 6(s(s^2 - 2) + 2), w = 6(s(s^2 - 2) - 2), p = 2(s^2 + 2)$$

Solutions from System III :

$$x = 6(2s^2 + 2s - 1), y = 6(2s^2 - 2s - 1),$$

$$z = 6(s(2s^2 - 1) + 2), w = 6(s(2s^2 - 1) - 2), p = 2(2s^2 + 1)$$

Solutions from System IV :

$$x = 66s, y = 18s,$$

$$z = 6(14s^2 + 2), w = 6(14s^2 - 2), p = 18s$$

Solutions from System V :

$$x = 18s, y = -6s,$$

$$z = 6(s^2 + 2), w = 6(s^2 - 2), p = 6s$$

Set 3:

Write (3) as

$$u^2 + 8v^2 = q^2 * 1 \tag{4}$$

Assume

$$q = a^2 + 8b^2 \tag{5}$$

Write integer 1 on the R.H.S. of (4) as

$$1 = \frac{(1 + i\sqrt{8})(1 - i\sqrt{8})}{9} \tag{6}$$

Using (5) and (6) in (3) and employing the method of factorization, define

$$(u + i\sqrt{8}v) = \frac{1}{3}(1 + i\sqrt{8})(a + i\sqrt{8}b)^2$$

On equating real and imaginary parts in the above equation and replacing a by $3A$ & b by $3B$,we get

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$$u = 3(A^2 - 8B^2 - 16AB)$$
$$v = 3(A^2 - 8B^2 + 2AB)$$
$$q = 9(A^2 + 8B^2)$$

In view of (2) , the non-zero distinct integer solutions of (1) are obtained as

$$x = 18(3A^2 - 24B^2 - 12AB) \, , y = 18(-A^2 + 8B^2 - 20AB) \, ,$$
$$z = 54[(A^2 - 8B^2)^2 - 14AB(A^2 - 8B^2) - 32A^2B^2] + 12 \, ,$$
$$w = 54[(A^2 - 8B^2)^2 - 14AB(A^2 - 8B^2) - 32A^2B^2] + 12 \, ,$$
$$p = 18(A^2 + 8B^2)$$

Note 1 :

Apart from (6) ,the integer 1 on the R.H.S. of (4) is also expressed as the product of complex conjugates as below:

$$1 = \frac{(7 + i2\sqrt{8})(7 - i2\sqrt{8})}{81} , 1 = \frac{(7 + i3\sqrt{8})(7 - i3\sqrt{8})}{121} ,$$
$$1 = \frac{(1 + i6\sqrt{8})(1 - i6\sqrt{8})}{289}$$

The repetition of the above process leads to three more sets of integer solutions to (1).

Set 4 :

Rewrite (3) as

$$q^2 - 8v^2 = u^2 * 1 \quad (7)$$

Write the integer 1 on the R.H.S. of (7) as

$$1 = (3 + \sqrt{8})(3 - \sqrt{8}) \quad (8)$$

Assume

$$u = a^2 - 8b^2 = (a + \sqrt{8b})(a - \sqrt{8b}) \quad (9)$$

Using (8) and (9) in (7) and using the method of factorization, define

$$q + \sqrt{8}v = (a + \sqrt{8b})^2 (3 + \sqrt{8})$$

Equating the coefficients of rational and irrational parts in (9) ,we get

$$q = 3(a^2 + 8b^2) + 16ab \, , v = (a^2 + 8b^2) + 6ab \quad (10)$$

Substituting the values of u, v, q from (9) and (10) in (2) , the non-zero distinct integral solutions of (1) are obtained as

$$x = 6(3a^2 + 8b^2 + 12ab)$$
$$y = 6(-a^2 - 24b^2 - 12ab)$$
$$z = 6[a^4 - 64b^4 + 6ab(a^2 - 8b^2) + 2]$$
$$w = 6[a^4 - 64b^4 + 6ab(a^2 - 8b^2) - 2]$$
$$p = 6(a^2 + 8b^2) + 32ab$$

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Note 2 :

It is worth to mention here that in (7), the integer 1 on the R.H.S. of (7) may also represented as follows

$$1 = \frac{\left(2s^2 + 1 + \sqrt{8}s\right)\left(2s^2 + 1 - \sqrt{8}s\right)}{\left(2s^2 - 1\right)^2}$$

Following the analysis as that of Set 4 , one may obtain different set of integer solutions to (1) .

II. CONCLUSION

As the bi-quadratic equations are rich in variety ,one may search for integer solutions to other choices of homogeneous or non-homogeneous bi-quadratic equations with multi-variables.

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