

Contribution of Linear Algebra and Matrix in Mathematics: An Analytical Study

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Abstract: *In this we are presenting a study on the linear algebra and matrix in mathematics. Linear Algebra is the branch of mathematics concerned with the study of vectors, vector spaces also called linear spaces, linear maps also called linear transformations, and systems of linear equations. Vector spaces are a central theme in modern mathematics: thus, linear algebra is widely used in both abstract algebra and functional analysis. Linear algebra also has a concrete representation in analytic geometry and it is generalized in operator theory. It has applications in the natural sciences and the social sciences. Since nonlinear models can often be approximated by linear ones..*

Keywords: Linear Algebra, Matrix, Linear Spaces, n-Tuples, Vectors, Linear Equation

I. INTRODUCTION

Linear algebra had its beginnings in the study of vectors in Cartesian 2-space and 3-space. A vector, here, is a directed line segment characterized by both its magnitude, represented by length, and its direction. Vectors can be used to represent physical entities such as forces, and they can be added to each other and multiplied with scalars, thus forming the first example of a real vector space. Modern linear algebra has been extended to consider spaces of arbitrary or infinite dimension.

A vector space of dimension n is called an n -space. Most of the useful results from 2- and 3-space can be extended to these higher dimensional spaces. Although people cannot easily visualize vectors in n -space, such vectors or n -tuples are useful in representing data. Since vectors, as n -tuples, are ordered lists of components, it is possible to summarize and manipulate data efficiently in this framework. For example, in economics, one can create and use, say, 8-dimensional vectors or 8-tuples to represent the Gross National Product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, and Australia). By using a vector $(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$ where each country's GNP is in its respective position.

A vector space or Linear space, as a purely abstract concept about which theorems are proved, is part of abstract algebra, and is well integrated into this discipline. Some striking examples of this are the group of invertible linear maps or matrices, and the ring of linear maps of a vector space. Linear algebra also plays an important part in analysis, notably, in the description of higher order derivatives in vector analysis and the study of tensor products and alternating maps. In this abstract setting, the scalars with which an element of a vector space can be multiplied not be numbers.

The only requirement is that the scalars form a mathematical structure, called a field. In applications, this field is usually the field of real numbers or the field of complex numbers. Linear maps take elements from a linear space to another or to itself, in a manner that is compatible with the addition and scalar multiplication given on the vector space. The set of all such transformations is itself a vector space. If a basis for a vector space is fixed. Every linear transform can be represented by a table of numbers called a matrix. The detailed study of the properties of and algorithms acting on matrices, including determinants and eigenvectors, is considered to be part of linear algebra. One can say quite simply that the linear problems of mathematics - those that exhibit linearity in their behavior - are those most likely to be solved. For example differential calculus does a great with linear approximation to functions. The difference from nonlinear problems is very important in practice. The general method of finding a linear way to look at

a problematic, expressing this in terms of linear algebra, and solving it, if need be by matrix calculations, is one of the most generally applicable in mathematics.

II. LINEAR ALGEBRA

A line passing through the origin (blue, thick) in \mathbb{R}^2 is a linear subspace, a common object of study in linear algebra. Linear algebra is a branch of mathematics concerned with the study of vectors, vector spaces (also called linear spaces), linear maps (also called linear transformations), and systems of linear equations. Vector spaces are a central theme in modern mathematics; thus. Linear algebra is widely used matrix, consult the invertible matrix article.

PRELIMINARY INTRODUCTION

Linear algebra had its beginnings in the study of vectors in Cartesian 2-space and 3-space. A vector, here, is a directed line segment, characterized by both its magnitude (also called length or norm) and its direction. The zero vector is an exception; it has zero magnitude and no direction. Vectors can be used to represent physical entities such as forces, and they can be added to each other and multiplied by scalars, thus forming the first example of a real vector space, where a distinction is made between "scalars", in this case real numbers, and "vectors". Modern linear algebra has been extended to consider spaces of arbitrary or infinite dimension. A vector space of dimension n is called an n -space. Most of the useful results from 2- and 3-space can be extended to these higher dimensional spaces.

Although people cannot easily visualize vectors in n -space, such vectors or n -tuples are useful in representing data. Since vectors, as n -tuples, consist of n ordered components, data can be efficiently summarized and manipulated in this framework. For example, in economics, one can create and use, say, 8-dimensional vectors or 8-tuples to represent the gross national product of 8 countries. One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, Armenia, Germany, Brazil, India, Japan, Bangladesh), by using a vector $(V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8)$ where each country's GNP is in its respective position.

USEFUL THEOREMS

Cardinality, equivalently, the dimension of a vector space is well-defined. A matrix is invertible if and only if its determinant is nonzero. A matrix is invertible if and only if the linear map represented by the matrix is an isomorphism. If a square matrix has a left inverse or a right inverse then it is invertible (see invertible matrix for other equivalent statements). A matrix is positive semi definite if and only if each of its Eigen values is greater than or equal to zero. A matrix is positive definite if and only if each of its Eigen values is greater than zero. An $N \times A$ matrix is diagonalizable (i.e. there exists an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$) if and only if it has n linearly independent eigenvectors.

LINEAR EQUATION

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and the first power of a single variable. Linear equations can have one or more variables. Linear equations occur abundantly in most subareas of mathematics and especially are particularly useful since many non-linear equations may be reduced to linear equations by assuming that quantities of interest vary to only a small extent from some "background" state. Linear equations do not include exponents. This article considers the case of a complete solution and, more generally for linear equations with coefficients and solutions in any field.

MATRIX

In mathematics, a matrix (plural matrices, or less commonly matrixes) is a rectangular array of numbers, as shown at the right. Matrices consisting of only one column or row are called vectors, while higher-dimensional, e.g. three-dimensional, arrays of numbers are called tensors. Matrices can be added and subtracted entry wise, and multiplied according to a rule corresponding to composition of linear transformations.

These operations satisfy the usual identities, except that matrix multiplication is not commutative: the identity $AB=BA$ can fail. One use of matrices is to represent linear transformations, which are higher-dimensional analogs of linear functions of the form $f(x) = cx$, where c is a constant. Matrices can also keep track of the coefficients in a system of linear equations. For a square matrix, the determinant and inverse matrix (when it exists) govern the behavior of solutions to the corresponding system of linear equations, and Eigen values and eigenvectors provide insight into the geometry of the associated linear transformation.

Matrices find many applications. Physics makes use of them in various domains, for example in geometrical optics and matrix mechanics. The latter also led to studying in more detail matrices with an infinite number of rows and columns. Matrices encoding distances of knot points in a graph, such as cities connected by roads, are used in graph theory, and computer graphics use matrices to encode projections of three-dimensional space onto a two-dimensional screen. Matrix calculus generalizes classical analytical notions such as derivatives of functions or exponentials to matrices. The latter is a recurring need in solving ordinary differential equations. Serialism and dodecaphonism are musical movements of the 20th century that utilize a square mathematical matrix to determine the pattern of music intervals. Due to their widespread use, considerable effort has been made to develop efficient methods of matrix computing, particularly if the matrices are big. To this end, there are several matrix decomposition methods, which express matrices as products of other matrices with particular properties simplifying computations, both theoretically and practically. Sparse matrices, matrices consisting mostly of zeros, which occur, for example, in simulating mechanical experiments using the finite element method, often allow for more specifically tailored algorithms performing these tasks. The close relationship of matrices with linear transformations makes the former a key notion of linear algebra. Other types of entries, such as elements in more general mathematical fields or even rings are also used.

III. CONCLUSION

Linear transformations and the associated symmetries play a key role in modern physics. Chemistry makes use of matrices in various ways, particularly since the use of quantum theory to discuss molecular bonding and spectroscopy. In this we are presenting a study on the linear algebra and matrix in mathematics. A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and the first power of a single variable. Linear equations can have one or more variables. Linear algebra is the branch of mathematics concerned with the study of vectors.

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