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The Cauchy Integral in Complex Analysis

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Abstract: *The study of analytic and harmonic functions forms a fundamental part of complex analysis, with applications ranging from physics to engineering. An analytic function is a complex function that is differentiable at every point in its domain, and a key property of such functions is that their real and imaginary parts are harmonic, satisfying Laplace's equation. This connection provides a powerful link between complex function theory and potential theory. However, the converse is not generally true: not every pair of harmonic functions corresponds to the real and imaginary parts of an analytic function [12,15].This discrepancy arises because analyticity imposes additional conditions, namely the Cauchy–Riemann equations, which harmonics alone may not satisfy. This work investigates the relationship between harmonic functions and analytic functions, exploring the conditions under which the former can be associated with the latter and highlighting cases where the converse fails. By analysing examples, counterexamples, and theoretical constraints, this study aims to clarify the subtle distinction between harmonicity and analyticity, thereby providing deeper insight into the structure of complex functions and their applications [11,14,15].*

Keywords: Analytic function, Harmonic function, Cauchy–Riemann equations, Complex differentiability, Necessary condition, Sufficient condition, Partial derivatives, Continuity

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