

A Review of Spectral Behavior of Sturm–Liouville Operators on Graphs and Networks

Amit¹ and Dr. Rishikant Agnihotri²

¹Research Scholar, Department of Mathematics

²Research Guide, Department of Mathematics
Kalinga University, Naya Raipur

Abstract: *Sturm–Liouville operators form a fundamental class of linear differential operators with wide applications in mathematical physics, engineering, and applied sciences. In recent decades, increasing attention has been devoted to the study of Sturm–Liouville problems defined on graphs and complex networks, motivated by applications in quantum mechanics, wave propagation, electrical circuits, and biological systems. The spectral behavior of these operators plays a crucial role in understanding stability, resonance phenomena, inverse problems, and dynamical processes on networks. This review paper presents a comprehensive overview of the spectral properties of Sturm–Liouville operators on metric graphs, focusing on eigenvalue distributions, boundary and matching conditions at vertices, self-adjoint realizations, and asymptotic behavior of spectra. Classical Sturm–Liouville theory is first recalled, followed by its extension to graph-based structures. Recent theoretical developments, spectral gaps, and applications are discussed, highlighting current challenges and future research directions.*

Keywords: Spectral Theory, Metric Graphs, Quantum Graphs, Boundary Conditions