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Solving High-Dimensional Partial Differential Equations using Deep Learning

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Abstract*: Solving high-dimensional partial differential equations (PDEs) poses a significant challenge due to the computational complexity and memory requirements involved. Traditional numerical methods encounter limitations when dealing with large-scale problems, motivating the exploration of alternative techniques. Deep learning has emerged as a promising approach to address these challenges by leveraging the representational power of neural networks. In the context of solving high-dimensional PDEs, deep learning techniques offer several advantages, including scalability, flexibility, and the ability to learn complex mappings between input and output spaces. By utilizing architectures such as convolutional neural networks (CNNs) and recurrent neural networks (RNNs), researchers have developed innovative methods to approximate solutions to high-dimensional PDEs. These approaches often involve training neural networks on simulated or experimental data to learn the underlying dynamics of the system, enabling efficient and accurate predictions. Additionally, techniques such as physics-informed neural networks (PINNs) integrate domain knowledge into the learning process, enhancing the robustness and interpretability of the models. Despite the progress achieved, challenges remain in optimizing network architectures, handling large datasets, and ensuring generalization to diverse problem domains. Nevertheless, the intersection of deep learning and high-dimensional PDEs holds great promise for advancing computational science and engineering applications, paving the way for more efficient and scalable solutions to complex physical phenomena*

Keywords: Deep Learning, Solving, Single-line Approach, Efficiency

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