IJARSCT

ISSN (Online) 2581-9429

International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)

International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal

Impact Factor: 7.301

Volume 3, Issue 3, December 2023

Deep Learning Approaches for High-Dimensional Partial Differential Equations Solution

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Abstract*: This abstract explores the application of deep learning techniques for solving high-dimensional partial differential equations (PDEs). High-dimensional PDEs pose significant challenges in traditional numerical methods due to the curse of dimensionality, making them computationally expensive and often infeasible. Deep learning, specifically neural networks, has shown promise in efficiently approximating complex functions and handling high-dimensional data. This paper reviews various deep learning approaches, such as convolutional neural networks (CNNs) and recurrent neural networks (RNNs), applied to the numerical solution of high-dimensional PDEs. The study discusses the advantages and limitations of these techniques, highlighting their potential to enhance accuracy and computational efficiency in comparison to classical methods. Additionally, it addresses the incorporation of domain knowledge and the exploration of hybrid methodologies to further improve the robustness and generalization of deep learning models in tackling challenging high-dimensional PDE problems.*

Keywords: Numerical Methods, Scientific Computing, Machine Learning

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