

Extension of some Polynomial Inequalities to the Polar Derivative and the Generalized Polar Derivative

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Abstract: In this paper certain polynomial inequalities for the polar derivative and the generalized polar derivative with restricted zeros are given, which generalize and refine some well-known polynomial inequalities due to Lax, Turán, AlSaeedi, Rather, Ali, Shafi, and Dar and others.

Keywords: Inequalities, restricted zeros, polynomials, polar derivative, generalized polar derivative.

REFERENCES

- [1]. T. Al - Saeedi. On Inequalities, Derivative and Maximum Modulus of Complex polynomial. D. Phil. Thesis, Dept. Math. J.N.V. University, Jodhpur (India), 2008.
- [2]. T. Al - Saeedi and D. M. Algawi. Integral Inequalities for the Polar Derivative and the Generalized Polar Derivative of Complex Polynomials, Univ. Aden J. Nat and Appl. Sc. Vol. 27, No. 2, 2023.
- [3]. Aziz and Q. M. Dawood. Inequalities for a Polynomial and its derivative, J. Approx. Theory, 1988.
- [4]. Aziz and N. A. Rather. Inequalities for the polar derivative of a polynomial with restricted zeros, Math. Balkanica 17, 15-28, 2003.
- [5]. Aziz and N. A. Rather. Some Zygmund type L^p Inequalities for polynomials, J. Math. Anal. Appl., 289, 14 - 29, 2004.
- [6]. S. Bernstein. Lecons sur les propriétés extrémales et la meilleure des fonctions réelles. Paris., 1926.
- [7]. R. P. Boas and Q. I. Rahman. L^p Inequalities for polynomials and entire functions Arch. Ration. Mach. Anal. 11, 34-39, 1962.
- [8]. P. Erdős. On extremal properties of derivatives of polynomials, Ann. Math. 41, 310-313, 1940.
- [9]. N. K. Govil and P. Kumar. On L^p Inequalities involving polar derivative of a polynomial. Acta. Math. Hung. 152, 130-139, 2017.
- [10]. N. K. Govil and Q. I. Rahman. Functions of exponential type not vanishing in a half - Plane and related polynomials. Transactions of the American Mathematical Society, 137, 501-517, 1969.
- [11]. E. Hille. Analytic Function Theory, Vol. II, Ginn and Company, New York, 1962.
- [12]. V. K. Jain. On polynomials having zeros in closed exterior or closed interior of a circle, Indian. J. Pure. Appl. Math. 30, 153-159, 1999.
- [13]. P. D. Lax. Proof of a conjecture of P. Erdős on the derivative of a polynomial, Bull. Amer. Mathematics Soc. 50, 509-513, 1944.
- [14]. M. A. Malik. An integral mean estimate for polynomials, Proc. Amer. Math. Soc. 91, 281-284, 1984.
- [15]. M. Marden. Geometry of Polynomial, Math. Survey No. 3, Amer. Math. Soc. Providence, RI, 1966.
- [16]. G. Polya and G. Szegő. Aufgaben und Lehrsätze aus der Analysis, Vol.1, p.137, problem 269, New York 1945.
- [17]. Q. I. Rahman, G. Schmeisser. L^p Inequalities for polynomials, J. Approx. Theory 53, 26 - 32, 1988.
- [18]. N. A. Rather, L. Ali, M. Shafi and I. Dar. Inequalities for the generalized polar derivative of a polynomial, Palestine Journal of Mathematics, Vol. 11 (3), 549-557, 2022.
- [19]. N.A. Rather, A. Iqbal and I. Dar. On the zeros of a class of generalized derivative, Rend. Circ. Mat. Palermo, II. Ser. 2020.
- [20]. P. Turán. Über die Ableitung Compositio Math. 7, 89 - 95, 1939.

[21]. A. Zygmund. A remark on conjugate series, Proc. Lond. Math. Soc. 341, 392 - 400, 1932.