

Review: Incorporating Nonlinear Dynamics Within Quantum Mechanics

Swati A. Raut¹

¹Department of Chemistry

Shriram Mahila Vindhyan Mahavidyalaya, Paniv, Malshiras, Solapur, (MS), India

swatiraut2022@Gmail.com

Abstract: *This research article embarks on a journey to unveil the often-unexplored intersections between nonlinear dynamics and quantum mechanics. Despite limitations in space, our investigation delves into two intriguing nonlinear parallels, prompting profound inquiries for readers. Employing a "quasi-botanical mode," we catalogue instances of nonlinear behaviour, aiming to provoke thought rather than furnish definitive quantitative validation. This perspective challenges the prevailing belief that quantum mechanics is inherently distinct from nature's inclination toward nonlinearities. In this endeavour, we echo Mielnik's notion of concealed "scientific newspeak," hinting at the presence of veiled nonlinear insights.*

Keywords: *quantum mechanics, nonlinear dynamics, intersections, critical inquiries, scientific newspeak.*

I. INTRODUCTION

The realm of quantum mechanics has long been synonymous with linearity, a core principle upheld by the Copenhagen interpretation championed by Bohr, Heisenberg, and their cohort. Yet, persistent challenges have arisen against this conventional viewpoint. These include Einstein's renowned objections in the Einstein-Bohr debates, the perplexing paradox embodied by Schrödinger's cat thought experiment, and the extensions of de Broglie and Bohm involving hidden variables. Nonetheless, the Copenhagen interpretation held its dominion throughout much of the 20th century, with only a sparse few delving into the fundamental underpinnings of quantum mechanics.

The emergence of quantum information science, and its potential implications for quantum computing, has engendered shifts in this landscape. In the last decade(s), a surge of both theoretical and experimental investigations has focused on Bell's theorem and inequalities. Notably, international workshops have been established to reexamine the very foundations of quantum mechanics and its apparent paradoxes. In parallel, attempts to expand quantum mechanics into the nonlinear realm have been proposed by various scholars, yet these endeavours have grappled with challenges, including the introduction of superluminal signals. This quandary is largely rooted in the fact that these nonlinear extensions tend to overlay nonlinear perturbations upon fundamentally linear systems. Only in the realm of "strongly nonlinear" systems can chaotic behaviour emerge a crucial element for comprehending the majority of quantum mechanical paradoxes. Mielnik encapsulates this dilemma, suggesting a potential avenue for resolution through a comprehensive reevaluation of traditional concepts.

In contrast to the above approaches, which can be termed "top-down," I have opted for a "bottom-up" strategy. I employ a "quasi-experimental" methodology, assembling specific instances where the principles of strong nonlinear and chaotic dynamics can mirror the enigmas, paradoxes, and inconsistencies that lie at the core of quantum mechanics, as generated by the Copenhagen interpretation. The initial examples, presented in descending order of clarity and increasing conjecture, include:

- The portrayal of exponential decay through unimodal maps.
- An alternative interpretation of Bell's inequalities utilizing nonlinear dynamics and nonextensive thermodynamics.
- Exploration of attractors, basins of attraction, and their implications for quantization.

- Investigation into spontaneous symmetry breaking, particularly parity no conservation.
- The study of decoherence and the transition from Hamiltonian to dispersive systems.
- Delving into the phenomenon of diffraction and the establishment of order within chaos.
- Understanding barrier penetration as a nonlinear occurrence.

Due to constraints of space, I will focus solely on the first two topics, which have garnered significant attention. However, a comprehensive record of my progress, including periodic revisions in thinking, can be found within a series of papers spanning several years [12, 13, 14, 15].

II. EXPLORING STATISTICAL EXPONENTIAL DECAY LAWS AND CHAOTIC ESCAPE FROM UNIMODAL MAPPINGS

Within the realm of quantum mechanics, a fascinating puzzle emerges: the emergence of exponential, first-order decay laws in processes like nuclear and particle decay, as well as atomic and molecular transitions. Quantum mechanics inherently asserts the indistinguishability of identical particles or systems. While it remains impossible to predict the precise moment of decay for a specific nucleus within a collection of radioactive nuclei, statistical significance emerges when dealing with a sufficiently large ensemble. Consequently, the decay of the collection adheres to a distinct exponential decay pattern. This is often likened, albeit inaccurately, to the statistical nature of actuarial tables in insurance, despite the contrasting origin of identical quantum states versus complex causal systems.

A more fitting analogy, however, can be drawn from the remarkable sensitivity of chaotic systems to initial conditions, aptly termed the "butterfly effect." Adhering to Ockham's razor principle, we seek the simplest mechanism capable of generating such behaviour. This leads us to the iteration of the quadratic (or logistic) map, a concept explored in depth in a previous study. For brevity, here we summarize the results.

The quadratic map is constructed through the iteration of the equation:

$$X_{n+1} = x_n^2 + C$$

In the latter equation, (x) represents a population ranging from extinction (0) to its maximum (1), and (A) denotes a birth rate parameter. Different ranges of (A) lead to varied outcomes: extinction, convergence to a single value of (x), bifurcation patterns with multiple alternating final values, and finally, chaos where infinitesimal differences in initial values yield unpredictable results. This chaotic behaviour is sensitive to initial conditions, except for intermittent windows of order within chaos.

An analogy is discerned between chaotic map iterations and radioactive decay. Imagine a slender interval of initial (x) values signifying the radioactive state(s), consistent with the Uncertainty Principle. Analogous to this, a slim interval in the final values denotes the concluding state. The iteration process symbolizes the decay process itself. For example, the number of attempts by an alpha particle to surmount the Coulomb barrier or the oscillations of a nuclear multipole leading to a gamma ray emission. This process, akin to decay, correlates to the number of iterations required for an initial value to "escape" into the final interval.

Through computer simulations, numerous maps have been tested, commencing with tens of thousands of randomly generated initial values within narrow intervals. These values venture into slightly broader final intervals. Remarkably, the tally of active trajectories (paths from each initial value) versus iteration count consistently yields statistical exponential decay curves. This pattern emerges across an array of unimodal maps, which represent smooth functions with a single peak and no inflection points. While this by no means constitutes conclusive evidence, it stands as the simplest and least convoluted explanation advanced thus far.

III. CLASSICAL VULNERABILITY IN BELL-TYPE INEQUALITIES

Ever since the publication of the EPR paradox [1], a persistent debate has revolved around the completeness of quantum mechanics and the potential breakdown of "local reality," a notion that led Einstein to coin the phrase "spooky action at a distance." The EPR paradox delves into correlations between conjugate variables, like the position and momentum of widely separated, non-communicating particles. It was designed to showcase the incompleteness of quantum mechanics. Initially conceptualized as an abstract Gedankenexperiment, David Bohm [16] later rendered EPR

more practical by investigating spin-1/2 fermions, and John Bell [17] then brought it closer to the realm of feasibility. Bell statistically treated correlations between distant particles and derived an inequality that sets an upper limit on specific classical statistical correlations. This framework allows quantum statistical correlations to transcend this upper bound under certain circumstances. While various refinements and versions of Bell's inequality exist, the CHSH inequality [18], designed with experimental tests in mind, serves as a straightforward example.

Enter Alice and Bob, standard figures in information theory. They are separated by a theoretically infinite distance, rendering them non-communicative. Correlated particle pairs, like electrons in a singlet state or photons with opposing polarizations, are prepared. One particle from each pair journeys to Alice, and the other reaches Bob, who independently makes binary measurements on their respective particles. Alice can choose to measure with respect to Q or R, yielding outcomes of +1 or -1. For instance, +1 could signify spin-up and -1 spin-down. Similarly, Bob's measurements correspond to S or T. These random choices occur independently, permitting Alice and Bob to measure using the same or different sets of axes. After an ample number of measurements to attain statistical significance, Alice and Bob compare notes using the quantity:

$$QS + RS + RT - QT = (Q + R)S + (R - Q)T$$

Given the independence of Q and R's values of +1 or -1, one of the terms on the right must equal 0. This leads to $(QS + RS + RT - QT = \pm 2)$, or in probability terms, where $(E(QS))$ represents the mean value of the QS measurement, the classical version of the CHSH inequality:

$$(E(QS) + E(RS) + E(RT) - E(QT) \leq 2)$$

This inequality imposes an upper limit on the possible statistical correlations that classical mechanics can achieve with specific combinations of measurements performed on widely separated particles.

On the quantum side, the derivation is analogous. However, the particle pairs are presumed to be in the entangled Bell singlet state:

$$(|\Psi\rangle) = \{ |01\rangle - |10\rangle / \sqrt{2} \}$$

The first qubit from each Ket goes to Alice, the second to Bob. Their measurements involve different combinations of observables:

$$Q = Z_1,$$

$$R = X_1$$

$$S = -\{ -Z_2 - X_2 \} / \sqrt{2}$$

$$T = \{ Z_2 - X_2 \} / \sqrt{2}$$

Here, (X) and (Z) correspond to quantum information matrices, akin to the Pauli spin matrices. Calculations show that the expectation values of QS, RS, and RT are all $(+1/\sqrt{2})$ while that of QT is $(-1/\sqrt{2})$. This leads to the CHSH quantum mechanical combination:

$$(E(QS) + E(RS) + E(RT) - E(QT) = 2\sqrt{2})$$

This value exceeds the classical inequality, suggesting that within the context of entangled systems, quantum mechanics can generate more significant statistical correlations than classical mechanics. Classical systems adhere to Bell-type inequalities, while quantum systems can at times violate them. Over recent decades, various Bell-type experiments have consistently favoured quantum mechanics. Interpretations may differ, but many entail the elimination of "local reality." Seemingly distant, isolated yet entangled particles influence each other. For instance, when Alice measures the spin direction of her electron, Bob's electron instantaneously aligns its spin in the opposite direction, suggesting real and superluminal action at a distance. Bell inequality physicists find themselves divided into two camps: those supporting Bell's theorem (which deems violation of the inequality incompatible with classical behaviour) and those who oppose it. Nonetheless, this paper proposes an alternative approach. It takes on the task of challenging a theorem that critics assert is irrelevant to many experimental results it has been applied to. This perspective offers a simpler alternative explanation and highlights a statistical misalignment often seen in quantum mechanics interpretations.

In this parallel, the issue arises not in quantum mechanics derivations but in classical ones. The classical correlations, deemed statistical, present in the initial particle pairs were subsequently disregarded, whereas the Bell entangled state inherently retained these correlations. This slip lies in the application of classical correlations that pertain to large

ensembles of particles but are mistakenly extended to individual measurements. This correlated behaviour in classical systems is familiar, requiring nonlinear behaviour, sometimes emergent in nature. Correlated statistics were formalized by Talis and colleagues [21, 22], introducing nonadditive thermodynamics. This approach relies on a generalized, nonextensive entropy. It reveals deviations from standard distributions when the exponent (q) differs from 1. This signifies the growing importance of "long-range" correlations. In classical systems with these correlations, the entropy becomes nonextensive, resulting in the total entropy of a two-component system ($A + B$) being greater (super extensive) or less (sub extensive) than the sum of individual entropies. Systems at the edge of quantum chaos [24] have demonstrated ($q > 1$), suggesting classical systems can exhibit long-range correlations. Classical behaviour marked by these correlations is not necessarily tied to long-range forces or action at a distance, as seen in cellular automata and emergent systems. This implies that the "classical" version of the CHSH inequality, and other forms of Bell's inequalities, warrant scepticism. Long-range correlations in classical systems can be constructive, potentially undermining the presumed upper limit. If values of (q) near 2 are applied, exponential rather than Gaussian statistical distributions might result. Consequently, Bell-type arguments might not effectively rule out the existence of "local reality" within quantum mechanics.

IV. CONCLUSION

Due to space constraints, delving into the various other nonlinear parallels isn't feasible. Nonetheless, the preceding sections exploring two such parallels should prompt thought-provoking inquiries within the reader's mind, even if definitive quantitative evidence is lacking. In essence, this paper aims to stimulate such questions rather than provide exhaustive quantitative validation. Our approach could be likened to a "quasi-botanical mode," involving the gathering and cataloguing of specimens, rather than an in-depth analysis. Nonlinear dynamics permeates virtually every scientific discipline, so why would quantum mechanics stand apart from nature's inclination towards feedback and nonlinearities? Perhaps, as Mielnik posited, we've unconsciously been adhering to a form of scientific "newspeak," inadvertently suppressing the expression of nonlinear "thoughtcrimes." The pioneers of early quantum mechanics operated without access to modern nonlinear dynamics and chaos theory. They grappled with the peculiarities of quantum mechanics within a strictly linear, and sometimes perturbative, framework. This approach thrived in terms of quantitative, real-world applications. Until recently, few questioned the validity of the Copenhagen interpretation or considered a possible breaking point where the application of increasingly esoteric concepts might falter. Who, after all, would be concerned about employing thirteenth-order perturbation theory [27] or 1078 parameters in variational calculations [28] to determine the ground-state energy of a helium atom with pinpoint precision? However, with the emergence of quantum computing on the horizon, perspectives are shifting. Quantum computing relies critically on linear superpositions of qubits. Although it remains uncertain whether quantum mechanics fundamentally entails nonlinear elements, this possibility alters the dynamics, particularly considering quantum computers. A nonlinear analogue to superposition might enable quantum computing, although this question remains unanswered for the future. What does seem plausible is that many interpretations of quantum mechanics have naively employed statistics. Statistics, including probabilities and wavefunctions, truly apply to large ensembles. Caution is essential when attributing single wavefunctions to single electrons and subsequently deriving specific expectation values.

In a final reflection, perhaps Einstein and Bohr were essentially correct in their debates. Chaos within quantum mechanics doesn't concern hidden variables; instead, it furnishes the determinism cherished by Einstein. Nevertheless, for practical intents and purposes, it yields uncertain outcomes necessitating statistical interpretation, as advocated by the Copenhagen interpretation. Contemplating how the Einstein-Bohr debates might have progressed with the availability of modern chaos theory offers intriguing speculation.

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