IJARSCT

ISSN (Online) 2581-9429

International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)

International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal

Impact Factor: 7.53

Volume 4, Issue 2, May 2024

The New Integral Transform "Bayawa Transform"

Zayyanu B. Bayawa[1] and Aisha A. Haliru[2]

Department of Mathematics, Federal College of Education, Gidan Madi, Sokoto, Nigeria[1]

Professor, Department of Mathematical Science, Bayero University Kano, Nigeria[2]

zayyanubalarabe01@gmail.com

Abstract: *In this paper a new integral transform namely Bayawa transform is proposed to solve linear ordinary differential equations with constant coefficients*

Keywords: Bayawa transform, Integral Transform and Differential Equations

I. INTRODUCTION

Integral transforms are tool used in solving linear ordinary differential equations, partial differential equations and integral equations among other applications as they are also used in control engineering applications. The integral transform of a function $f(x)$ defined in $a \le x \le b$ is denoted by $\mathfrak{F}\{f(x)\} = \mathrm{F}(k)$, and defined by

$$\mathfrak{F}\{f(x)\} = \mathrm{F}(k) = \int_a^b K(x,k)\ f(x)dx,$$

Where $K(x,k)$, given function of two variables x and k, is called the kernel of the transform. The operator $\mathfrak{F}$ is usually called an integral transform operator or simple an integral transformation. The transform function $\mathrm{F}(k)$ is often referred to as the image of the given object function $f(x)$ and k is called the transform variable. The idea of the integral transform operator is somewhat similar to that of the well known linear differential operator, $\boldsymbol{D} \equiv \frac{d}{dx}$ which acts on a function $f(x)$ to produce another function $f'(x)$, that is,

$$\boldsymbol{D}f(x) = f'(x).$$

Usually, $f'(x)$ is called derivative or the image of $f(x)$ under the linear transformation $\boldsymbol{D}$

Bayawa Integral transform has been introduced to facilitate the process of solving ordinary and partial differential equations in time domain, it has been derived from the classical Fourier integral. Joseph Fourier in 1822, Fourier treatise provided the modern mathematical theory of heat conduction, Fourier series and Fourier integral with applications. In his treatise, Fourier stated a remarkable result that is universally known as Fourier theorem. He give a series of examples before stating that an arbitrary function defined on a finite interval can be expanded in terms trigonometric series which is now universally known as the Fourier series. In an attempt to extend his new ideas to functions defined on an infinite interval, Fourier discovered an integral transform and its inversion formula which are now well known as Fourier transform and the inverse Fourier transform.

The oldest integral transform and also the most commonly used in the Laplace transform by P.S Laplace in 1780s, this has effectively been used in finding the solution of linear differential equations and integral equations. Others include Stieltjes transform, Melin transform, Hankel transform, Laguerre transform, Hilbert transform, Rodon transform and wavelet transform among others. Furthermore, of recent Samudu transform, Natural transform, Elzaki transform, Aboodh transform. Other recent integral transforms include the new integral transform developed by kashuri and fundo transform, **M** transform. The ZZ transform, Ramadan Group transform. Kamal transform. Mohand transform. Shehu transform, Sawi transform, Kharrat-Toma transform, Sohan transform, Emad -Faith transform, NE transform. Finally, Iman transform.

Are the convenient mathematical tools for solving differential equations, Also Bayawa Transform and some of its fundamental properties are used to solve differential equations.

Bayawa Integral Transform

The proposed Integral Transform is defined for an exponential order function:

$U = \{ f(t){:}\exists M,\ h_1 , h_2 >0.\ |f(t)| < Me^{-\frac{|t|}{h_j}} ,\text{if } t \in (-1)^j \times [0,\infty)\}$

For a given function in the set U the constant M must be finite number h_1, h_2 may be finite or infinite.

Bayawa Transform denoted by the operator B (.) defined by the integral equations.

$ß\{f(t)\} = Z(v) = v^2 \int_0^\infty f(t)\, e^{-\frac{t}{v^2}}\, dt, t \geq 0, h_1 \leq v \leq h_2$

The variable v is the transform is used to factor the variable t in the argument of the function f this transform has deeper connection with the Laplace, Sumudu, Elzaki, Kamal,Mohand,Mahgoub, kharrat- toma transforms etc. In this work we want to show the applicability of this interesting new transform and its efficiency in solving the linear differential equations.

1.1 Bayawa transform of the some functions

Bayawa integral transform, the existence condition must be met to apply the transform in to any function.

The existence condition to apply $Z[f(t)]$ to any function $f(t)$ is for $t \geq 0$ must be piecewise continuous and in exponential order.

Bayawa integral transform can use in solving the following functions.

(i) let $f(t) = 1$, then, by definition, we have

$$ß\,[1] = Z(v) = v^2 \int_0^\infty f(t)\, e^{-\frac{t}{v^2}} dt$$

$$= v^2 \int_0^\infty 1.\, e^{-\frac{t}{v^2}}\, dt$$

$ß\,[1] = v^4$

(ii) let $f(t) = t$, the

$ß[t] = v^2 \int_0^\infty t.\, e^{-\frac{t}{v^2}}\, dt$

Apply integral by part, we have

$ß[t] = v^6$

(iii) let $f(t =)\ t^2$, then $ß[\,t^2] = v^2 \int_0^\infty t^2.\, e^{-\frac{t}{v^2}}\, dt$

Apply integral by part, we have

$ß[\,t^2] = 2!\, v^8$

(iv)let $f(t) = t^3$, then

$ß\,[t^3] = v^2 \int_0^\infty t^3.\, e^{-\frac{t}{v^2}} dt$

IJARSCT
ISSN (Online) 2581-9429
International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
Impact Factor: 7.53
Volume 4, Issue 2, May 2024

Apply integral by part, we have

$ß[\,t^3] = 3!\,v^{10}$

(v) let $f(t) = t^n$, then

$ß[t^n\,] = v^2\int_0^\infty t^n.e^{-\frac{t}{v^2}}\,dt$

Let $x = \frac{t}{v^2}$ or $t = v^2x$, $dt = dxv^2$

$= v^2\int_0^\infty e^{-x}(v^2x)^n\,.v\,dx$, Since gamma n +1 $= \int_0^\infty e^{-x}(x)^n = n!$

$ß[t^n\,] = v^{2n+4}.n!$ Where n is an integer number

(vi) $ß[e^{at}\,] = v^2\int_0^\infty e^{at}.e^{-\frac{t}{v^2}}\,dx = v^2\int_0^\infty e^{-\left(\frac{1}{v^2}-a\right)t}\,dt$

$= \frac{v^4}{1-av^2}$

(vii) $ß[\,e^{-at}\,] = \frac{v^4}{1+av^2}$

$(viii)$ $ß[sinat] = v^2\int_0^\infty sinat.e^{-\frac{t}{v^2}}dt$

Since $sinat = \frac{e^{iat} - e^{-iat}}{2i}$

$= \frac{av^6}{1+a^2v^4}$

(ix) $ß[cosat] = v\int_0^\infty cosat.e^{-\frac{t}{v^2}}\,dt$

Since $cosat = \frac{e^{iat} - e^{-iat}}{2}$

$ß\,[cosat] = \frac{v^4}{1+a^2v^4}$

(x) $ß\,[sinhat] = \frac{av^6}{1-a^2v^4}$

(xi) $ß\,[coshat] = \frac{v^4}{1-a^2v^4}$

1. 2 Theorem: let $Z\,(v)$ is Bayawa integral transform of $f(t)$ denoted by $ß[\,f(t)]$ that is $[ß\{f(t)\} = Z\,(v)]$, then :

The integral transform "Bayawa"

(1) $ß[f'(t\,)] = \frac{1}{v^2}Z(v) - v^2f(0)$

(2) $ß[f''(t\,)] = \frac{1}{v^4}Z(v) - f(0) - v^2f'(0)$

(3) $ß[f^n(t\,)] = \frac{1}{v^{2n}}Z(v) - \sum_{k=0}^{n-1} v^{-2n+2k+4}f^{(k)}(0)$

Proof:

(1) $ß[f'(t\,)] = v^2\int_0^\infty f(t)\,e^{-\frac{t}{v^2}}dt$

Apply integration by part, we have

$ß[f'(t\,)] = \frac{1}{v^2}Z(v) - v^2f(0)$

(2) Let $g(t) = f'(t\,)$, then

$ß[g'(t\,)] = \frac{1}{v^2}ß[\,f'(t)] - v^2f'(0)$

DOI: 10.48175/IJARSCT-18119
ISSN 2581-9429 IJARSCT

We find that by using (i)

$ß[f''(t)] = \frac{1}{v^4}Z(v) - f(0) - v^2 f'(0)$

(3) Can be proof by Mathematical Induction

1.3 Application Bayawa transform in to ordinary differential equations

For linear systems that governed by differential equations, Bayawa Integral transform could be used as an efficient tool to analyze their basic characteristics in response into initial data.

The Efficiency Bayawa can be expressed in solving some certain initial value problems described by ordinary differential equations.

(i)Consider the linear first order ordinary differential equations

$$\frac{dy}{dt} + py = f(t)\ , t > 0, y(0) = a$$

Where a and p are constants and $f(t)$ is an external input function so that $ß[\boldsymbol{f(t)}]$transform exists.

Now apply Bayawa transform with initial condition

$$\frac{1}{v^2}Z(v) - v^2 f(0) + pZ(v) = \bar{f}(v)$$

$$\frac{1}{v^2}Z(v) - av^2 + pZ(v) = \bar{f}(v)$$

$$Z(v)\left[\frac{1}{v^2} + p\right] = \bar{f}(v) + a$$

$$Z(v)\left[\frac{1+pv^2}{v^2}\right] = \bar{f}(v) + av^2$$

$$Z(v) = \frac{v^2(\bar{f}(v) + av^2)}{1 + pv^2}$$

$$Z(v) = \frac{v^2\bar{f}(v)}{1 + pv^2} + \frac{av^4}{1 + pv^2}$$

The Inverse Bayawa Transform give the solution

(ii)Consider the linear second order ordinary differential equations

For a second order ordinary differential equation that has the general form:

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x), (0) = a, y'(0) = b, a\ , b, p \text{ and } q \text{ are constants}$$

Apply Bayawa transform with the initial value problem $a\ , b, p$ and q would give:

$$\frac{1}{v^4}Z(v) - f(0) - v^2 f'(0) + 2p\left(\frac{1}{v^2}Z(v) - v^2 f(0)\right) + qZ(v) = \bar{f}(v)$$

$$\frac{1}{v^4}Z(v) - a - bv^2 - \frac{2p}{v^2}Z(v) - 2apv^2 + qZ(v) = \bar{f}(v)$$

$$Z(v)\left[\frac{1}{v^4} + \frac{2p}{v^2} + \text{q}\right] = a + bv^2 + 2apv^2 + \bar{f}(v)$$

$$Z(v)\left[\frac{1+2pv^2+qv^4}{v^4}\right] = a + bv^2 + 2apv^2 + \bar{f}(v)$$

$$Z(v) = \frac{v^4(a + bv^2 + 2av^2p + \bar{f}(v))}{1 + 2v^2 p + qv^4}$$

$$Z(v) = \frac{v^4 a + bv^6 + 2av^6 p + v^4\bar{f}(v))}{1 + 2v^2 p + qv^4}$$

$$Z(v) = \frac{v^4\bar{f}(v) + bv^6 + v^4 a(1 + 2av^2 p)}{1 + 2v^2 p + qv^4}$$

$Z(v) = \frac{v^4 \bar{f}(v)}{1+2v^2\,p+qv^4} + \frac{bv^6}{1+2v^2\,p+qv^4}\,\frac{v^4 a(\,1+2av^2 p)}{1+2v^2\,p+qv^4}$

The inverse Bayawa transform gives the solution

Example 1

Consider the first order ordinary differential equations

$\frac{dy}{dx} + y = 0, y(0) = 1$

Apply Bayawa transform with initial condition

$\frac{1}{v^2} Z(v) - v^2 f(0) + Z(v) = 0$

Where $Z(v)$ is the Bayawa transform of the function $y(x)$

$\frac{1}{v^2} Z(v) - v^2(1) + Z(v) = 0$

$Z(v)\,[\,\frac{1}{v^2} + 1\,] = v^2$

$Z(v)\,[\,\frac{1+v^2}{v^2}\,] = v^2$

$Z(v) = \frac{v^4}{1+v^2}$

Take inverse Bayawa transform, we have

$y(x) = e^{-x}$

Example 2

Consider the first order ordinary differential equation

$\frac{dy}{dx} + 2y = 0, y(0) = 1$

Apply Bayawa transform with initial condition

$\frac{1}{v^2} Z(v) - v^2 f(0) + 2Z(v) = v^6$

Where $Z(v)$ is the Bayawa transform of the function $y(x)$

$\frac{1}{v^2} Z(v) - v^2(1) + 2Z(v) = v^6$

$Z(v)\,[\,\frac{1}{v^2} + 2\,] = v^6 + v^2$

$Z(v) = \frac{v^2(v^6 + v^2)}{2v^2 + 1}$

Take synthetic division method and inverse Bayawa Transform, we have

$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$

Example 3

Consider the first order ordinary differential equation

$\frac{dy}{dx} + y = 3, y(0) = 1$

Apply Bayawa transform with initial condition

$\frac{1}{v^2}Z(v) - v^2 f(0) + Z(v) = 3v^4$

Where $Z(v)$ is the Bayawa transform of the function $y(x)$

$\frac{1}{v^2}Z(v) - v^2 f(1) + Z(v) = 3v^4$

$Z(v)\left[\frac{1}{v^2} + 1\right] = 3v^4 + v^2$

$Z(v) = v^2\left(\frac{3v^4 + v^2}{1 + v^4}\right)$

Take synthetic division method and inverse Bayawa Transform

$y(x) = 3 - 2e^{-x}$

Example 4

Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + y = 0 \; y(0) = 1, y'(0) = 1$

Apply Bayawa transform with initial condition

$\frac{1}{v^4}Z(v) - f(0) - v^2 f'(0) + Z(v) = 0$ $\frac{1}{v^4}Z(v) -$

$1 - v^2 + Z(v) = 0$

$Z(v)\left[\frac{1}{v^4} + 1\right] = 1 + v^2$

$Z(v)\left[\frac{1 + v^4}{v^4}\right] = 1 + v^2$

$Z(v) = \frac{v^4(1 + v^2)}{1 + v^4}$

$Z(v) = \frac{v^4}{1 + v^4} + \frac{v^6}{1 + v^4}$

Take inverse Bayawa transform, we have

$y(x) = sinx + cosx$

Example 5

Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0 \; y(0) = 1, y'(0) = 4$

Now apply Bayawa transform with initial condition

$\frac{1}{v^4}Z(v) - f(0) - v^2 f'(0) - 3\left(\frac{1}{v^2}Z(v) - v^2 f(0)\right) + 2Z(v) = 0$

$\frac{1}{v^4}Z(v) - 1 - 4v^2 - \frac{3}{v^2}Z(v) + 3v^2 + 2Z(v) = 0$

$Z(v)\left[\frac{1}{v^4} - \frac{3}{v^2} + 2\right] = 1 + 4v^2 - 3v^2$

$Z(v) = v^4\left(\frac{1 + v^2}{1 - 3v + 2v^4}\right)$

Now, $\frac{1 + v^2}{1 - 3v + 2v^4} = \frac{1 + v^2}{(1 - 2v^2)(1 - v^2)} = \frac{A}{1 - 2v^2} + \frac{B}{1 - v^2}$

Therefore $A = 3, B = -2$

$Z(v) = v^2[\frac{3}{1-2v^2} - \frac{2}{1-v^2}]$

Take inverse Bayawa transform, we have

$y(x) = -2e^x + 3e^{2x}$

Example 6

Consider the second order ordinary differential equation

$\frac{d^2y}{dx^2} + 4y = 12t\ y(0) = 0, y'(0) = 7$

Now apply Bayawa transform with initial condition

$\frac{1}{v^4}Z(v) - f(0) - v^2 f'(0) + 4Z(v) = 12v^6$

$\frac{1}{v^4}Z(v) - 0 - 7v^2 + 4Z(v) = 12v^6$

$Z(v)\ [\frac{1}{v^2} + 4] = 12v^6 + 7v^2$

$Z(v)\ = \frac{12v^{10} + 7v^4}{4v^4 + 1}$

Take synthetic division method and inverse Bayawa Transform

$Z(v) = 3v^4 + \frac{4v^4}{4v^4 + 1}$

$y(x) = 3x + 2sin2x$

II. CONCLUSION

In this papers, we have proposed, A New Integral Transform Bayawa Transform for solving ODE we have demonstrated the applicability and have shown new efficient the method is in solving ODE.

III. ACKNOWLEDGEMENT

An upmost appreciation and been grateful to the Prof. Aisha Abubakar Haliru who provided me with all the necessary guidance and her intellectual methodology and psychological perception, managed to guide me to the completion of this research work, and eventually getting the sense of direction of what exactly i expecting, particularly on this research work, may Allah guide her to the right path and reward her abundantly. Ameen

REFERENCES

[1]. G.K watugala, sumudu transform –a new integral transform to solve differential equation and control engineering problem.Math.Engrginduct.6(1998),no 4,319-329

[2]. TarigM.Elzaki(2011), The New Integral Transform “Alzaki Transform” Global journal of pure and applied Mathematics ISSN 0973-1768,N0.1,pp.57-64

[3]. K.S Aboodh, The New Integral Transform “Aboodh Transform” Global Journal of Pure and Applied mathematics,9(1),35-43(2013)

[4]. AbdelilahKamal.H.Sedeeg The New Integral Transform “ Kamal Transform”, Advance in Theoretical and Applied Mathematics,vol.11,no.2(2016),pp.451-458

[5]. Mohand. M.AbderahimMahgoub .The New Integral Transform "Mahgoub Transform" Advance in Theoretical and Applied Mathematics ISSN:0973-4554 vol.11,No.4(2016).pp.391-398

[6]. Shehu M, Weidng Z, New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations,International Journal of Analysis and Application.Vol.17,no.2(2019),167-190.

[7]. D.P.Patil, SavitaSantuKhakale, The New Integral Transform "Sohan Transform". International Journal of Advance in Engineering and Management (IJAEM),vol.3,issue 10,10(2021),pp:126-132,ISSN:2395-5252

[8]. Emadkuffi, Sara faith Maktoof, "Emad – Faith transform" a new integral transform. Journal of Interdisciplinary Mathematics ISSN: 0972-05029(print),ISSN:2169-12x(online),vol.24(2021),No.6 pp.2381-2390.

[9]. Iman Ahmed Almardy, The New Integral Transform "Iman Transform" International journal of Advanced Research in Science, Communication and Technology(IJARSCT)vol.3,1ssue1,3(2023)

[10]. Stieltjes. T. S, Recherches sur les fraction continues , Annales de la At e des Science de Toulouse, 18 (1894).1-123

[11]. H. Mellin, $\hat{A}I\acute{U}$ber die fundamentale Wichtgkeit des Satzes von Cauchy f$\hat{A}Iur$ die theorien der Gamma- un der hypergeometrics funktionen, Acta Soc.Fennicae, 21(1896) 1-115

[12]. J. Fourier, La Th$\hat{A}\grave{t}$eorie Analytique de la chalerur (1822), English Translation by A. Dover publications (1955)

[13]. D. Hilbert, Grubndz$\hat{A}\grave{I}$uge einer allgemeinen Theorie der linearen Integralgeichungen (1912) 75-77

[14]. J. Rodon, $\hat{A}I\acute{U}$ber dieBestimmung von Funktionen durch Integralw I $\hat{A}Ia\acute{r}te$ I$\hat{A}Ia\acute{n}gs$gewisser manningfaltikreiten, Ber Verh. Akad. Wiss.Math. –Nat ., Leipzig, 69, (1917) 26-277

[15]. L. Debnath, D.Bhatta, Integral transforms and its applications , Third Edition, CRC press Tailor and Francis Group, Boca Raton London New York. (2015) .

[16]. A. Grossman and J. morlet, Decomposition of Hardy functions into square integrable wavelets of constants shape, SIAMJ.Math.Anal ., 15, (1984) 723-736

[17]. Z.H. Khan and W.A.Khan , Natural transform –properties applications, NUST Journal of Engineering Science, 1(2008) 127-133

[18]. A. kashuri and A.fundo,"A New Integral Transform", Advance in Theoretical and Applied Mathematics,vol.8,no.1,2013,pp.27-43

[19]. H.M. Srivastava, A new integral transform and its applications, Acta Mathematica Scientia, 35B(6), (2015) 1386-1400

[20]. Z.Zafar, ZZ transform method , International Journal of Advanced Engineering and Global Technology, 04(01) 2016

[21]. R.K. Raslan, M.A Ramadan , T.S.El- Danaf and A.R. Hadhoud, On a new general integral transform: some properties and remarks, J.Math.Comput.Sci., 23(1) , (2016) 103-109